

Divisibility Tests.

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Differentiate and Assess

Not every student will be ready to investigate this concept at this Level and so we will need to differentiate to ensure every student is learning at their leading edge. Select the Differentiate button on this screen.

Integrate

Every mathematical concept is integrally related to other mathematical concepts. Teaching and learning related concepts simultaneously develops deep relational understanding. Select the Integrate button on this screen.

Intervene

Some students may not yet be ready to investigate this concept at any Level, and so we will need to provide some intervention. Select the Intervention button on this screen.

DIVISIBILITY TESTS.

EXPLICIT TEACHING PLAN OVERVIEW PAGE

THIS PAGE IS A SUMMARY OF THE EXPLICIT TEACHING PLAN, INCLUDING STRATEGIC QUESTIONS, AND DESCRIBING THE SEQUENCE WHICH WILL OCCUR OVER MULTIPLE LESSONS.

RESOURCES: PLAYING CARDS,PENCIL, PAPER

| WHAT COULD WE DO? | WHAT LANGUAGE COULD WE USE TO EXPLAIN AND ASK QUESTIONS? |
|---|---|
| <p>Children:</p> <ul style="list-style-type: none">• Test numbers for divisibility by 2 by identifying it is even.• Test numbers for divisibility by 4 by identifying if it is even, whether it has an odd tens digit and ones digits 2 and 6 or an even tens digit and ones digits 4, 8 and zero.• Test numbers for divisibility by 8 by identifying if it is even, whether it has an even hundreds digit and tens and ones digits that are multiples of 8, or an odd hundreds digit and tens and ones digits that are multiples of 8 plus 4.• Test numbers for divisibility by 3 by identifying whether the digits add to make a multiple of 3.• Test numbers for divisibility by 9 by identifying whether the digits add to make a multiple of 9.• Test numbers for divisibility by 6 by testing for divisibility by 2 and 3.• Test numbers for divisibility by 10 by identifying if the ones digit is zero.• Test numbers for divisibility by 5 by identifying if the ones digit is zero or 5. | <p>Children</p> <ul style="list-style-type: none">• ask one another questions about divisibility, such as;<ul style="list-style-type: none">• If all even numbers are multiples of 2, is 2 a factor of all even numbers? Why?• If we know that every hundreds number is divisible by 4, do we just need to look at the ones and tens numbers to test if a number is divisible by 4? Why?• If every thousands number is divisible by 8, do we just need to look at the ones and tens and hundreds numbers to test if a number is divisible by 8? Why?• If the sum of the digits is divisible by 3, will the number also be divisible by 3? Why?• If the sum of the digits is divisible by 9, will the number also be divisible by 9? Why?• If a number is divisible by both 2 and 3, will it also be divisible by 6? Why?• Do numbers that are divisible by 10 have zero ones because of place value? Why?• Do numbers that are divisible by 5 have zero or 5 ones because of place value? Why? |

DIVISIBILITY TESTS.

EXPLICIT TEACHING PLAN

FULL EXPLICIT TEACHING PLAN, EMBEDDING DEEP RELATIONAL UNDERSTANDING, METALANGUAGE, AND QUESTIONS THAT MAY BE USED OVER MULTIPLE LESSONS.

Comment [CS1]:

| WHAT COULD WE DO? | WHAT LANGUAGE COULD WE USE TO EXPLAIN AND ASK QUESTIONS? |
|---|---|
| Children think about, talk and listen to a friend about, then have the opportunity to share what they already know. | <ul style="list-style-type: none">▶ Today brings an investigation about divisibility.▶ What do you know about divisibility?▶ Talk about divisibility with a friend.▶ Is anyone ready to share what they are thinking about divisibility? |
| Record, for example, divisibility | <ul style="list-style-type: none">▶ Today we're going to investigate divisibility.▶ What do you think divisibility means?▶ Do you think it has something to do with division?▶ If a number is divisible by another number, what does that mean?▶ Does divisible mean we get a whole number as the quotient when we divide one number by another number? |
| Record, for example, divisible | <ul style="list-style-type: none">▶ Are number divisible by their factors?▶ Let's investigate how we can tell if a number is divisible by another number without doing the division. |
| Record, for example, divisibility by 2 | <ul style="list-style-type: none">▶ Let's start with divisibility by 2.▶ How do we know if a number is divisible by 2?▶ We've investigated multiples of 2.▶ And we found that all even numbers are multiples of 2. |

Record, for example, even numbers are divisible by 2

Record, for example, odd numbers are not divisible by 2.

Record, for example, divisibility by 4

Allow children to suggest some multiples of 4.

Record, for example, odd numbers are not divisible by 4

- ▶ Since all even numbers are multiples of 2, then all even numbers are divisible by 2.
- ▶ If all even numbers are multiples of 2, is 2 a factor of all even numbers?
- ▶ Since all even numbers are multiples of 2, then 2 is a factor of all even numbers!

- ▶ What about odd numbers?
- ▶ Are odd numbers multiples of 2?
- ▶ Since odd numbers are not multiples of 2, then odd numbers are not divisible by 2.
- ▶ If odd numbers are not multiples of 2, is 2 a factor of any odd numbers?
- ▶ Since odd numbers are not multiples of 2, then 2 is not a factor of any odd numbers!

- ▶ **Let's investigate divisibility by 4.**
- ▶ How do we know if a number is divisible by 4?
- ▶ Think of some multiples of 4.
- ▶ Are any odd numbers divisible by 4?
- ▶ So no odd numbers are divisible by 4.
- ▶ Is every even number divisible by 4?
- ▶ Is 2 divisible by 4?
- ▶ No, 2 is not divisible by 4.
- ▶ Is 4 divisible by 4?
- ▶ Yes, 4 is divisible by 4.
- ▶ Is 6 divisible by 4?

Record, for example, every second even number is divisible by 4

Record, for example, 100 is divisible by 4

Record, for example, every hundreds number is divisible by 4

Record, for example, 2500 is divisible by 4

Record, for example, 35 700 is divisible by 4

Record, for example, every hundreds number is divisible by 4

Record, for example, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40

- ▶ No, 6 is not divisible by 4.
- ▶ Is 8 divisible by 4?
- ▶ Yes, 8 is divisible by 4.
- ▶ So is every second even number divisible by 4?

- ▶ So we know that every second even number is divisible by 4, but how will we know whether a high even number is a second even number?
- ▶ Let's investigate hundreds.
- ▶ Is 100 divisible by 4?
- ▶ If 100 is divisible by 4, are multiples of 100 divisible by 4?
- ▶ Is every hundreds number divisible by 4?
- ▶ What about 25 hundred?
- ▶ Is 25 hundreds, 2 thousand 5 hundred?
- ▶ What about 357 hundred?
- ▶ Is 357 hundred, 35 thousand 7 hundred?
- ▶ So is every hundreds number divisibility by 4?

- ▶ If we know that every hundreds number is divisible by 4, do we just need to look at the ones and tens numbers to test if a number is divisible by 4?

- ▶ We know every second even number is divisible by 4. Which 2 digit numbers are these?
- ▶ Let's look at the first 10 multiples of 4 to see if we can identify any patterns: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
- ▶ In numbers with an even tens digit, what are the ones digits?

Underline the ones digits in numbers with an even number of tens digits, for example, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40

Underline the ones digits in numbers with an even number of tens digits, for example, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40

Record, for example, 56

Record, for example, 356

Record, for example, 7356

Record, for example, divisibility by 8

Allow children to suggest some multiples of 8, for example, 8, 16, 24.

Record, for example, odd numbers are not divisible by 8

- ▶ Is zero an even number?
- ▶ We've investigated zero, and we found it is an even number because we can halve zero and get a whole number, and zero comes before an odd number.
- ▶ So in numbers with an even tens digit, are the ones digits 4, 8 and zero?
- ▶ In numbers with an odd tens digit, what are the ones digits?
- ▶ In numbers with an odd tens digit, are the ones digits 2 and 6?

- ▶ Is 56 divisible by 4?
- ▶ Yes, 56 is divisible by 4 because there is an odd number of tens and the ones digit is 6.
- ▶ Is 356 divisible by 4?
- ▶ Yes, 356 is divisible by 4 because every hundreds number is divisible by 4, there is an odd number of tens and the ones digit is 6.
- ▶ Is 7356 divisible by 4?
- ▶ Yes, 7356 is divisible by 4 because every hundreds number is divisible by 4, there is an odd number of tens and the ones digit is 6.

- ▶ **Let's investigate divisibility by 8.**
- ▶ How do we know if a number is divisible by 8?
- ▶ Think of some multiples of 8.
- ▶ Are any odd numbers divisible by 8?
- ▶ No, odd numbers are not divisible by 8.
- ▶ So all of the multiples of 8 are even numbers.
- ▶ Is every even number divisible by 8?

Record, for example, not every even number is divisible by 8

- ▶ No, not every even number is divisible by 8.
- ▶ How can we tell if an even number is divisible by 8?
- ▶ Let's investigate whether hundreds numbers are divisible by 8.
- ▶ Is 100 divisible by 8?
- ▶ Is 100, 80 plus 20?
- ▶ No, 100 is not divisible by 8, we'll have 4 remaining.
- ▶ Is 200 divisible by 8?
- ▶ Is 200, 160 plus 40?
- ▶ Yes, 200 is divisible by 8.

Record, for example, every even hundreds number is divisible by 8

- ▶ If 200 is divisible by 8, are multiples of 200 divisible by 8?
- ▶ Is every even number of hundreds number divisible by 8?
- ▶ What about 1 thousand, 6 hundred?
- ▶ Is 1 thousand, 6 hundred, 16 hundred?
- ▶ What about 34 thousand, 6 hundred?
- ▶ Is 34 thousand, 6 hundred, 346 hundred?

Allow children to calculate if 1000 is divisible by 8, for example,

- ▶ Is 1000 divisible by 8?
- ▶ Is 1000 an even number of hundreds?
- ▶ Is 1000, 800 plus 200?
- ▶ So 1000 is divisible by 8.
- ▶ If 1 thousand is divisible by 8, are multiples of 1000 divisible by 8?
- ▶ Is every thousands number divisible by 8.
- ▶ Is 26 thousand divisible by 8?

Record, for example, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96

Record, for example, 64 is divisible by 8

Record, for example, 264 is divisible by 8

Record, for example, 864 is divisible by 8

Record, for example, all numbers with an even hundreds digit will be divisible by 8 if their two-digit number is divisible by 8.

- ▶ Is 1 265 thousand divisible by 8? Is 1 265 thousand, 1 million, 265 thousand?
- ▶ If we know that every thousands number is divisible by 8, will we know whether a number is divisible by 8 just by looking at the ones, tens and hundreds digits?
- ▶ So we know that every even hundreds number is divisibility by 8.
- ▶ How could we use this information to test any number's for divisibility by 8?
- ▶ Let's look at the two-digit multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96
- ▶ How many hundreds in these numbers?
- ▶ Are there zero hundreds?
- ▶ Is zero an even number?
- ▶ We've investigated zero, and we found it is an even number because we can halve zero and get a whole number, and zero comes before an odd number.
- ▶ So does a number with zero hundred have an even number of hundreds?
- ▶ In all numbers with an even hundreds digit, will multiples of 8 be the same as the two-digit multiples?
- ▶ If 64 is divisible by 8, will 264 also be divisible by 8?
- ▶ Is 200 divisible by 8?
- ▶ Because 200 is divisible by 8, will 200 plus a two-digit multiple of 8 also be divisible by 8?
- ▶ Will 864 also be divisible by 8?
- ▶ Will all numbers with an even hundreds digit be divisible by 8 if their two-digit number is divisible by 8?

Record, for example, 104 is divisible by 8

Record, for example, Numbers with an odd number of hundreds will be divisible by 8 if their two-digit number is 4 less and 4 more than a two-digit number that is divisible by 8

- ▶ What about numbers with an odd hundreds digit?
- ▶ We know 96 is divisible by 8.
- ▶ So 104 must also be divisible by 8.
- ▶ 104 has an odd hundreds number.
- ▶ If 104 is divisible by 8, is 304 divisible by 8?
- ▶ 504?
- ▶ 704?
- ▶ 1904?
- ▶ 27 304?
- ▶ Is 4, 4 less than 8?
- ▶ What will the next multiple of 8 be after 104?
- ▶ Will 112 be the next multiple of 8?
- ▶ Is 12, 4 less than 16?
- ▶ Is 12 also 4 more than 8?
- ▶ So numbers with an odd number of hundreds will be divisible by 8 if their two-digit number is 4 less and 4 more than a two-digit number that is divisible by 8?
- ▶ Is 152 divisible by 8?
- ▶ Is 52, 4 less than 56? Is 56 divisible by 8?
- ▶ Is 52, 4 more than 48? Is 48 divisible by 8?

Record, for example, 10 is not divisible by 3

Record, for example, 9 is divisible by 3


Record, for example, 100 is not divisible by 3

Record, for example, 99 is divisible by 3

Record, for example, 1000 is not divisible by 3

Record, for example, 999 is divisible by 3

Record a place value column, for example,



Point to the values, identifying that 999 is one less than a thousand, 99 is one less than 100, and 9 is one less than 10.

Record, for example, 1 less than every place value is divisible by 3.

Record, for example, 258

Record, for example, $258 = 200 + 50 + 8$

► **Let's investigate divisibility by 3.**

► How do we know if a number is divisible by 3?

► Is 10 divisible by 3?

► No, 10 is not divisible by 3, but 9 is!

► Is 100 divisible by 3?

► No, 100 is not divisible by 3, but 99 is! Is 1000 divisible by 3?

► No, 1000 is not divisible by 3, but 999 is!

► So 10 is not divisible by 3, but 9 is and 100 is not divisible by 3, but 99 is, and 1000 is not divisible by 3, but 999 is.

► So 1 less than 10, 100 and 1000, is divisible by 3.

► Have you noticed that these are the values of our place value columns?

► And that one less than the value of each place value column is divisible by 3!

► Let's investigate how we can use our understanding of place value to test if a number is divisible by 3.

► Let's test if 258 is divisible by 3 using our understanding of place value.

► What does 258 mean in standard place value?

► Is 258, 200, plus 50, plus 8.

$$200 - (99 + 99) = 2$$

$$50 - (9 + 9 + 9 + 9 + 9) = 5$$

$$8 = 8$$

Record, for example, $2 + 5 + 8 = 15$

Record, for example, 15 is divisible by 3, so 258 is divisible by 3

Point to the original number, for example, 258

Record, for example, if the sum of the digits is divisible by 3, the number will also be divisible by 3

Record, for example, 564

Record, for example, $5 + 6 + 4 = 15$

Record, for example, 15 is divisible by 3, so 564 is divisible by 3

Record, for example, 86

Record, for example, $8 + 6 = 14$

- ▶ Let's look at the 200.
- ▶ From the 200, we know that out of each hundred, 99 is divisible by 3, so that leaves 2
- ▶ Let's look at the 50.
- ▶ From the 50, we know that out of each 10, 9 is divisible by 3, so that leaves 5
- ▶ And then we also have the 8.

- ▶ So that leaves the 2, the 5 and the 8.
- ▶ If $2 + 5 + 8$ is divisible by 3, then 258 is divisible by 3.
- ▶ What does 2 plus 5 plus 8 equal?
- ▶ Does 2 plus 5 plus 8 equal 15?
- ▶ 15 is divisible by 3, so 258 is divisible by 3.

- ▶ Did you notice that the numbers we were left with were the same as the digits in the original number?
- ▶ So if the sum of the digits is divisible by 3, will the number also be divisible by 3?

- ▶ Is 564 divisible by 3?
- ▶ What does 5 plus 6 plus 4 equal? Does 5 plus 6 plus 4 equal 15?
- ▶ Is 15 divisible by 3?
- ▶ So because the sum of the digits is divisible by 3, is 564 divisible by 3?
- ▶ Is 86 divisible by 3?
- ▶ What does 8 plus 6 equal? Does 8 plus 6 equal 14?
- ▶ Is 14 divisible by 3?
- ▶ So because the sum of the digits is not divisible by 3, is 86 also not divisible by

Record, for example, 14 is not divisible by 3, so 564 is not divisible by 3

Record, for example, 10 is not divisible by 9

Record, for example, 9 is divisible by 9

Record, for example, 100 is not divisible by 9

Record, for example, 99 is divisible by 9

Record, for example, 1000 is not divisible by 9

Record, for example, 999 is divisible by 9

Record a place value column, for example,



Point to the values, identifying that 999 is one less than a thousand, 99 is one less than 100, and 9 is one less than 10.

Record, for example, 1 less than every place value is divisible by 9.

3?

► **Let's investigate divisibility by 9.**

► How do we know if a number is divisible by 9?

► Is 10 divisible by 9?

► No, 10 is not divisible by 9, but 9 is!

► Is 100 divisible by 9?

► No, 100 is not divisible by 9, but 99 is!

► Is 1000 divisible by 9?

► No, 1000 is not divisible by 9, but 999 is!

► So 10 is not divisible by 9, but 9 is and 100 is not divisible by 9, but 99 is, and 1000 is not divisible by 9, but 999 is.

► So 1 less than 10, 100 and 1000, is divisible by 9.

► Have you noticed that these are the values of our place value columns?

► And that one less than the value of each place value column is divisible by 9!

► Let's investigate how we can use our understanding of place value to test if a number is divisible by 9.

Record, for example, 468

Record, for example, $468 = 400 + 60 + 8$

$$400 = \underbrace{99 + 99 + 99 + 99}_4 + 2$$
$$60 = \underbrace{9 + 9 + 9 + 9 + 9 + 9}_6 + 6$$

$$8 = 8$$

Record, for example, $4 + 6 + 8 = 18$

Record, for example, 18 is divisible by 9, so 468 is divisible by 9

Point to the original number, for example, 468

Record, for example, If the sum of the digits is divisible by 9, the number will also be divisible by 9

Record, for example, 564

Record, for example, $5 + 6 + 4 = 15$

Record, for example, 15 is not divisible by 9, so 564 is not divisible by 9

- ▶ So let's check if 468 is divisible by 9 using our understanding of place value. 468 means 400, plus 60, plus 8.
- ▶ Let's look at the 400.
- ▶ From the 400, we know that out of each hundred, 99 is divisible by 9, so that leaves 4.
- ▶ Let's look at the 60.
- ▶ From the 60, we know that out of each 10, 9 is divisible by 9, so that leaves 6
- ▶ And then we also have the 8.
- ▶ So that leaves the 4, the 6 and the 8.
- ▶ If $4 + 6 + 8$ is divisible by 9, then 468 is divisible by 9.
- ▶ What does 4 plus 6 plus 8 equal?
- ▶ Does 4 plus 6 plus 8 equal 18?
- ▶ 18 is divisible by 9, so 468 is divisible by 9.
- ▶ Did you notice that the numbers we were left with were the same digits in the original number?
- ▶ So if the sum of the digits is divisible by 9, will the number also be divisible by 9?

- ▶ Is 564 divisible by 9?
- ▶ What does 5 plus 6 plus 4 equal? Does 5 plus 6 plus 4 equal 15?
- ▶ Is 15 divisible by 9?
- ▶ So because the sum of the digits is not divisible by 9, is 564 also not divisible by 9?

- ▶ **Let's investigate divisibility by 6.**

Record, for example, If a number is divisible by both 2 and 3, it will also be divisible by 6.

Record, for example, 84

Record, for example, 84 is divisible by 2

Record, for example, 84 is divisible by 3

Record, for example, 84 is divisible by 6

Record, for example, 364

Record, for example, 364 is divisible by 2

Record, for example, 364 is not divisible not divisible by 3

Record, for example, 364 is not divisible by 6

- ▶ How do we know if a number is divisible by 6?
- ▶ What are the factors of 6?
- ▶ Is 2 a factor of 6?
- ▶ Is 3 a factor of 6?
- ▶ Because we multiply 2 by 3 to get 6, if a number is divisible by both 2 and 3, will it also be divisible by 6?
- ▶ Is 84 divisible by 2?
- ▶ Is 84 an even number?
- ▶ Yes, 84 is divisible by 2.
- ▶ Is 84 divisible by 3?
- ▶ Does 8 plus 4 equal 12?
- ▶ Is 12 divisible by 3?
- ▶ Yes, 84 is divisible by 3.
- ▶ Because 84 is divisible by 2 and by 3, is 84 also divisible by 6?
- ▶ Will there be half as many 6s in 84 as there are 3s?
- ▶ Will there be a third as many 6s in 84 as there are 2s?
- ▶ If we want to multiply by 6, could we multiply by 3 and then double?
- ▶ If we want to divide by 6, could we divide by 3 and then halve?
- ▶ Is 364 divisible by 6?
- ▶ Is 364 divisible by 2?
- ▶ Is 364 divisible by 3? Does $3 + 6 + 4$ add to make a multiple of 3?

- ▶ So if 364 is divisible by 2, but not divisible by 3, is 364 not divisible by 6?
- ▶ **Let's investigate divisibility by 10.**

Record, for example, 360 is divisible by 10

Record, for example, 364 is not divisible by 10

Record, for example, 360 is divisible by 5

Record, for example, 365 is divisible by 5

- ▶ How do we know if a number is divisible by 10?
- ▶ What digit do all numbers that are divisible by 10 have in the ones column?
- ▶ Why?
- ▶ Do numbers that are divisible by 10 have zero ones because in our place value system, we multiply and divide by 10?
- ▶ So if a number has a zero in the ones place, it is divisible by 10?
- ▶ Is 360 divisible by 10?
- ▶ Does 360 have zero in the ones place?
- ▶ Is 364 divisible by 10?
- ▶ Does 364 have zero in the ones place?

▶ **Let's investigate divisibility by 5.**

- ▶ How do we know if a number is divisible by 5?
- ▶ What digits do all numbers that are divisible by 5 have in the ones column?
- ▶ Do numbers that are divisible by 5 have zero ones or 5 ones?
- ▶ Why?
- ▶ Do numbers that are divisible by 5 have zero ones or 5 ones because in our place value system, we multiply by 10 and 5 is a factor of 10?
- ▶ Is every second multiple of 5 a tens number?
- ▶ So if a number has a zero or 5 in the ones place, it is divisible by 5?
- ▶ Is 360 divisible by 5?
- ▶ Does 360 have a zero or 5 in the ones place?
- ▶ Is 365 divisible by 5?
- ▶ Does 365 have a zero or 5 in the ones place?
- ▶ Is 364 divisible by 5?

Record, for example, 364 is not divisible by 5

► Does 364 have a zero or 5 in the ones place?