

MULTIPLICATION AND DIVISION BY 9.

INVESTIGATIONS OVERVIEW PAGE

THIS PAGE IS A SUMMARY OF THE INVESTIGATIONS THAT STUDENTS MAY ENGAGE IN TO DEEPEN THEIR RELATIONAL UNDERSTANDING. INVESTIGATIONS WITH INSTRUCTIONS TO STUDENTS FOLLOW ON SUBSEQUENT PAGES.

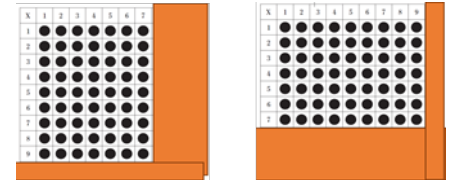
- Children sit with a friend. They select cards to make numbers to multiply by 9. They multiply their number by 9 using the distributive property and / or relationships between multiplication by 9 and multiplication by 10, and by multiplication by 3 and by 3. They record their multiplication number sentence. If they multiplied a single-digit number by 9, they learn the multiple of 9 (table). They explain to a friend how they multiplied the number by 9. **Reflection:** How can we use the distributive property and relationships to multiplication by other numbers, to multiply by 9?
- Children sit with a friend. They select cards to make numbers to divide by 9 / ninth. They divide their number by 9 by partitioning it into multiples of 9 that they do know, and / or relationships between division by 9 and division by 10, and division by 3 and by 3. They record their division number sentence. They explain to a friend how they divided by 9 / ninth. **Reflection:** How can we divide by 9 / ninth?
- Children use a calculator to multiply numbers by 9, then check using the distributive property. Children explain their strategy to a friend. **Reflection:** How can we use the distributive property to multiply by 9?
- Children use a calculator to divide numbers by 9, then check by partitioning it into multiples of 9 that they do know. Children explain their strategy to a friend. **Reflection:** How can we divide by 9 / ninth?
- In pairs, each child has 2 cards. Each child places their cards face down and records each number's product when multiplied by 9 on paper next to the pair, for example, Children tell their friend what number they think is on the card by saying 'I think the number is 7 because 7 times 9 equals 63'. If correct, the child takes the card. **Reflection:** How can we use the distributive property to multiply by 9?
- In pairs, each child has 2 cards. Each child places their cards face down and records each number's product when multiplied by 9 on paper next to the pair, for example, Children tell their friend what number they think is on the card by saying 'I think the number is 7 because 63 divided by 9 equals 7'. If correct, the child takes the card. **Reflection:** How can we divide by 9 / ninth?
- In pairs, children lay out 5 cards in a row. They take turns to secretly select a card and to name a number that, when divided by 9, gives that number as the quotient. If correct, they keep the card. For example, they may secretly select 6 and say, 'When I divide 54 by 9, the quotient is this number', or secretly select 4 and 1 to make 14, and say, 'When I divide 126 by 9, the quotient is this number'. **Reflection:** How can we multiply and divide by 9?

<input type="text"/>	63
<input type="text"/>	45

<input type="text"/>	63
<input type="text"/>	45

- In pairs, children play 'guess my number'. They take it in turns to describe a number after it has been multiplied by 9. For example, they may secretly select 8 and say, 'When my number is multiplied by 9, the product is 72' or you may secretly select 4 and 1 to make 14 and say, 'When my number is multiplied by 9, the product is 126' **Reflection: How can we multiply and divide by 9?**
- Children flip cards to make single-digit numbers to multiply by 9. They record the multiplication number sentence. They use the commutative property to record another multiplication sentence. They then use the inverse relationship between multiplication and division to record 2 division number sentences. For example, they flip 6 and record $6 \times 9 = 54$ and $9 \times 6 = 54$. They then record $54 \div 9 = 6$ and $54 \div 6 = 9$. **Reflection: How can we multiply and divide by 9?**

Children have a multiplication by single-digit numbers array and use it to recall multiples of 9 through properties and relationships. This includes seeing the number of rows and the number in each row multiplicatively, and breaking the number of rows into smaller numbers of rows to allow for subitising. For example, 9×7 could be represented using 2 sheets of card, to allow children to subitise, 9 rows of 5 plus 9 rows of 2, and to see 9×7 commutatively as 7×9 .



NB: This array replaces the traditional multiplication table chart where products are in the place of the dots.

Any multiplication table charts with products listed actually ensure children don't learn to recall single-digit multiplication because it is easier to look at the product on the chart than to use properties and relationships to work out a single-digit multiplication. **Reflection: How can we multiply and divide by 9?**

- **To focus on learning multiples of 9 (tables)**, children flip cards one at a time and name the product when the number is multiplied by 9. If the child knows the product when multiplied by 9 efficiently (quite quickly, and without much conscious thought), they place the card in the 'know' pile. If the child does not know the product when multiplied by 9 efficiently, they place the card in the 'learn' pile.

Once they have 2 cards with different numbers in the 'learn' pile, the child stops flipping cards.

They look at the numbers that they cannot yet multiply by 9, and consider some strategies to assist them to efficiently multiply the number by 9.

If they cannot think of any strategies themselves, they ask their classmates in either a structured sharing session, or while investigating multiplying and dividing by 9.

They record the multiplication facts, and related division facts, for example,

$$6 \times 9 = 54 \qquad 9 \times 6 = 54$$

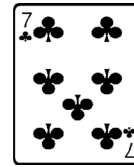
$$54 \div 9 = 6 \qquad 54 \div 6 = 9$$

Reflection: How can we multiply and divide by 9?

Multiplication And Division By 9

Select cards to make a number to multiply by 9, for example,

Multiply the number by 9 using the distributive property, and / or relationships between multiplication by 9 and multiplication by 10, and by multiplication by 3 and by 3, for example,



$9 \times 7 = 63$ $\begin{array}{c} \diagup \quad \diagdown \\ 5 \quad + \quad 2 \end{array}$ $9 \times 5 = 45$ $9 \times 2 = 18$ $45 + 18 = 63$	$9 \times 7 = 63$ $10 \times 7 = 70$ $70 - 7 = 63$	$9 \times 7 = 63$ $3 \times 7 = 21$ $3 \times 21 = 63$
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Record the product, for example, $9 \times 7 = 63$

If you multiplied a single-digit number by 9, learn the multiple of 9 (table).

Explain to a friend how you multiplied the number by 9.

Reflection: How can we use the distributive property and relationships to multiplication by other numbers, to multiply by 9?

Multiplication And Division By 9

Select cards to make a number to divide by 9 / ninth, for example,



or



Divide your number by 9 by partitioning it into multiples of 9 that you do know, and / or relationships between division by 9 and division by 10, and division by 3 and by 3, for example,

$$72 \div 9 = 8$$

$$\begin{array}{l} 27 + 45 \end{array}$$

$$27 \div 9 = 3$$

$$45 \div 9 = 3$$

$$5 + 3 = 8$$

$$\frac{1}{9} \text{ of } 72 = 8$$

$$\begin{array}{l} 27 + 45 \end{array}$$

$$\frac{1}{9} \text{ of } 27 = 3$$

$$\frac{1}{9} \text{ of } 45 = 5$$

$$5 + 3 = 8$$

$$72 \div 3 = 24$$

$$24 \div 3 = 8$$

$$71 \div 9 = 7 \text{ r}8$$

$$\begin{array}{l} 27 + 44 \\ 36 + 8 \end{array}$$

$$27 \div 9 = 3$$

$$36 \div 9 = 4$$

$$3 + 4 = 7$$

$$\frac{1}{9} \text{ of } 71 = 7 \text{ r}8$$

$$\begin{array}{l} 27 + 44 \\ 36 + 8 \end{array}$$

$$\frac{1}{9} \text{ of } 27 = 3$$

$$\frac{1}{9} \text{ of } 36 = 4$$

$$3 + 4 = 7$$

Record the quotient.

Explain to a friend how you divided by 9 / ninthed.

Reflection: How can we divide by 9 / ninth?

Multiplication And Division By 9

Use a calculator to multiply numbers by 9.

Check using the distributive property.

Explain your strategy to a friend.

Reflection: How can we use the distributive property to multiply by 9?

Multiplication And Division By 9

Use a calculator to divide numbers by 9 / ninth.

Check by partitioning into multiples of 9 that you know.

Explain your strategy to a friend.

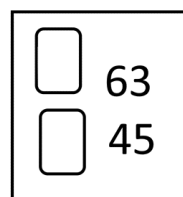
Reflection: How can we divide by 9 / ninth?

Multiplication And Division By 9

Sit with a friend.

Each of you has 2 cards.

Place your cards face down and record each number's product when multiplied by 9, on paper next to the card, for example,



Take turns to tell your friend what number you think is on each card, explaining using multiplication, by saying 'I think the number on the card is 7 because 9 times 7 equals 63'.

If correct, you take the card.

Reflection: How can we use the distributive property to multiply by 9?

Multiplication And Division By 9

Sit with a friend.

Each of you has 2 cards.

Place your cards face down and record each number's product when multiplied by 9 on paper next to the card, for example,

<input type="text"/>	63
<input type="text"/>	45

Take turns to tell your friend what number you think is on each card, explaining using division, by saying 'I think the number on the card is 7 because 63 divided by 9 equals 7'.

If correct, you take the card.

Reflection: How can we divide by 9 / ninth?

Multiplication And Division By 9

Sit with a friend.

Lay out 5 cards in a row.

Take turns to secretly select cards and to name a number that, when divided by 5, gives that number as the quotient.

If your friend works out the quotient, they keep the card.

For example, you may secretly select 6 and say, 'When I divide 54 by 9, the quotient is this number'.

For example, you may secretly select 4 and 1 to make 14, and say, 'When I divide 126 by 9, the quotient is this number'.

Reflection: How can we multiply and divide by 9?

Multiplication And Division By 9

Sit with a friend.

Lay out 5 cards in a row.

Take turns to secretly select cards.

Describe the number after it has been multiplied by 5.

For example, you may secretly select 8 and say, 'When my number is multiplied by 9, the product is 72'.

For example, you may secretly select 4 and 1 to make 14 and say, 'When my number is multiplied by 9, the product is 126'.

Reflection: How can we multiply and divide by 9?

Multiplication And Division By 9

Select a card to make single-digit numbers to multiply by 9.

Record the multiplication number sentence.

Use the commutative property to record another multiplication sentence.

Use the inverse relationship between multiplication and division to record 2 division number sentences.

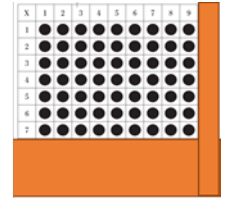
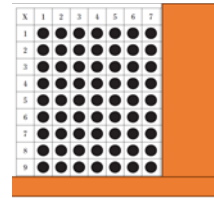
For example, flip 6 and record $6 \times 9 = 54$ and $9 \times 6 = 54$. Then record $54 \div 9 = 6$ and $54 \div 6 = 9$.

Reflection: How can we multiply and divide by 9?

Multiplication And Division By 9

Have a multiplication by single-digit numbers array and use it to recall multiples of 4 (tables!) through properties and relationships. This includes seeing the number of rows and the number in each row multiplicatively, and breaking the number of rows into smaller numbers of rows to allow for subitising.

For example, 9×7 could be represented using 2 sheets of card, to allow children to subitise, 9 rows of 5 plus 9 rows of 2, and to see 9×7 commutatively as 7×9 .



Reflection: How can we multiply and divide by 9?

X	1	2	3	4	5	6	7	8	9	10
1	●	●	●	●	●	●	●	●	●	●
2	●	●	●	●	●	●	●	●	●	●
3	●	●	●	●	●	●	●	●	●	●
4	●	●	●	●	●	●	●	●	●	●
5	●	●	●	●	●	●	●	●	●	●
6	●	●	●	●	●	●	●	●	●	●
7	●	●	●	●	●	●	●	●	●	●
8	●	●	●	●	●	●	●	●	●	●
9	●	●	●	●	●	●	●	●	●	●
10	●	●	●	●	●	●	●	●	●	●

Multiplication And Division By 9

To focus on learning multiples of 9:

Flip cards one at a time and name the product when the number is multiplied by 9.

If you know the product when multiplied by 9 efficiently (quite quickly, without skip count), place the card in the 'know' pile.

If you do not know the product when multiplied by 9 efficiently, place the card in the 'learn' pile.

Once you have 2 cards with different numbers in the 'learn' pile, stop flipping cards.

Look at the numbers that you cannot yet multiply by 9, and use multiplicative place value to assist you to efficiently multiply the number by 9.

If you cannot think of any strategies yourself, ask your classmates in either a structured sharing session, or while investigating multiplying and dividing by 9.

Record the multiplication facts, and related division facts, for example,

$$6 \times 9 = 54$$

$$9 \times 6 = 54$$

$$54 \div 9 = 6$$

$$54 \div 9 = 3$$

Reflection: How can we multiply and divide by 9?