

Volume, Capacity, Displacement.

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Differentiate and Assess

Not every student will be ready to investigate this concept at this Level and so we will need to differentiate to ensure every student is learning at their leading edge.

Integrate

Every mathematical concept is integrally related to other mathematical concepts. Teaching and learning related concepts simultaneously develops deep relational understanding.

Intervene

Some students may not yet be ready to investigate this concept at any Level, and so we will need to provide some intervention.

VOLUME, CAPACITY, DISPLACEMENT.

EXPLICIT TEACHING PLAN OVERVIEW PAGE

THIS PAGE IS A SUMMARY OF THE EXPLICIT TEACHING PLAN, INCLUDING STRATEGIC QUESTIONS, AND DESCRIBING THE SEQUENCE WHICH WILL OCCUR OVER MULTIPLE LESSONS.

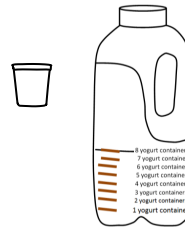
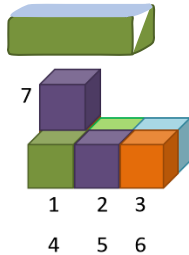
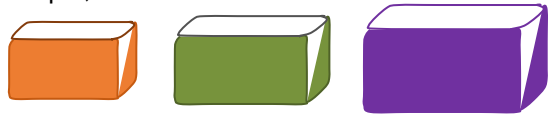
RESOURCES: CUBES, SMALL CONTAINERS AND SPOONS, 2L MILK OR JUICE BOTTLES, PERMANENT MARKERS, WATER, CONTAINERS OF VARIOUS SIZES, FOR EXAMPLE LUNCHBOXES, PLASTICINE, ROCKS AND STONES, PENCIL, PAPER

WHAT COULD WE DO?

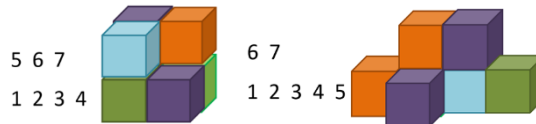
WHAT LANGUAGE COULD WE USE TO EXPLAIN AND ASK QUESTIONS?

Children:

- make a measuring device using informal liquid units, for example,
- use the measuring device to measure, compare and order the volume and capacity of containers with curved surfaces, for example,



- create models using cubes, measuring volume by counting, for example,



- create different models using the same number of cubes, explaining the volumes are the same, for example,

- investigate the volume of objects with curved surfaces using displacement, for example,



- compare the volumes of objects with curved surfaces using displacement, for example,

Children

- ask one another questions about volumes of models in cubes, liquid informal units, including displacement, for example:
 - ▶ how could we make a measuring device to measure the volume and capacity of containers with curved surfaces?
 - ▶ how could we use the measuring device to measure the volume and capacity of containers with curved surfaces?
 - ▶ how could we make a model using cubes?
 - ▶ how could we count the cubes to find the model's volume?
 - ▶ how could we make different models with the same volume?
- how could we use displacement to measure the volume of curved objects?

VOLUME, CAPACITY, DISPLACEMENT.

EXPLICIT TEACHING PLAN

FULL EXPLICIT TEACHING PLAN, EMBEDDING DEEP RELATIONAL UNDERSTANDING, METALANGUAGE, AND QUESTIONS THAT MAY BE USED OVER MULTIPLE LESSONS.

WHAT COULD WE DO?

Children think about, talk and listen to a friend about, then have the opportunity to share what they already know.

WHAT LANGUAGE COULD WE USE TO EXPLAIN AND ASK QUESTIONS?

- ▶ Today brings an investigation about volume and capacity.
- ▶ What do you know about volume and capacity?
- ▶ Talk about volume and capacity with a friend.
- ▶ Is anyone ready to share what they are thinking about volume and capacity?

- ▶ We've investigated dimensions.
- ▶ And we found that up and down is one dimension.
- ▶ We found that left to right is another dimension.
- ▶ And we found that front to back is another dimension.

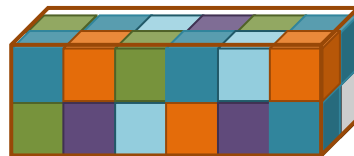
- ▶ We've investigated volume.
- ▶ And we found that volume is the amount of space an object takes up in all three dimensions.

- ▶ We've investigated capacity.
- ▶ And we found that capacity is the volume a container can hold when it is filled to capacity.

- ▶ We've investigated measuring volume and capacity.
- ▶ And we found that if the surfaces of a hollow object are thin, the volume and capacity will be about the same.

- ▶ We found that we could use cubes to measure the volume and capacity of objects with flat surfaces and straight lines.

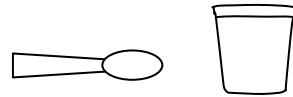
Display a prism filled with cubes packed in layers, for example,



Display a cylinder filled with water, for example,



Display a small yogurt container and spoon, for example,



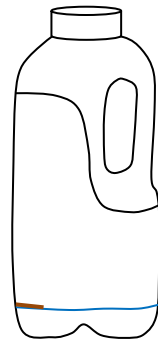
Display a container, for example, a two litre milk or juice container, for example,



Display a small container, for example, a yogurt container



Fill the yogurt container with water and pour it into the milk container, for example,



Mark the height of the water and label the mark '1 yogurt container'

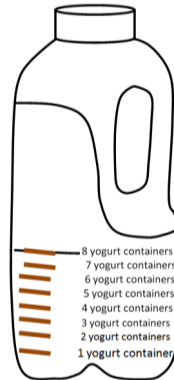
- ▶ We found that a cube is the best object to measure the volume or capacity of prisms because the cube takes up the same space when we change its orientation.
- ▶ We found that we could use units of water to measure the volume and capacity of objects with curved surfaces and curved lines because water leaves no gaps.
- ▶ We measured the volume and capacity of curved objects using water by pouring in cupfuls or spoonfuls of water and counting the number of cupfuls or spoonfuls
- ▶ Could we find an easier way to measure volume and capacity with water without having to continually count cupfuls or spoonfuls? Let's investigate!
- ▶ What is this object?
- ▶ Do you think we could make a measuring device out of this milk container?
- ▶ What could we use as our unit of measurement?
- ▶ Could we use this yogurt container?
- ▶ Let's fill the yogurt container with water and pour it into the milk container.
- ▶ Let's make a mark where the water comes up to on the milk container.
- ▶ Let's name the mark with our unit of measurement - 1 yogurt container.

Fill the yogurt container with water and pour it into the milk container again, for example,

Mark the height of the water and label the mark '2 yogurt containers'

Continue filling the yogurt container with water, pouring it into the milk container, marking and labelling each new water level, for example,

Display an object with curved surfaces and lines, for example, a lunchbox



- ▶ Let's fill the yogurt container with water and pour it into the milk container again.
- ▶ Let's make a mark where the water comes up to on the milk container now.
- ▶ Let's name the mark with our unit of measurement - 2 yogurt containers.

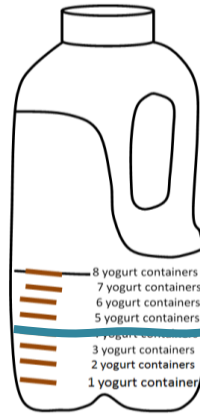
- ▶ Let's continue filling the yogurt container with water, pouring it into the milk container, marking and labelling each new water level.

- ▶ Now that we have made a measuring device, could we use it to measure the volume and capacity of containers?
- ▶ How could we use our measuring device to measure the volume and capacity of this lunchbox?
- ▶ Because the surfaces of the lunchbox are thin, will its volume and capacity be almost the same?
- ▶ Does this lunchbox have slightly curved surfaces and curved lines?
- ▶ Could we measure the volume and capacity of the lunchbox using cubes?
- ▶ Will the slight curve on the surfaces and lines create gaps if we pack it in layers with cubes?

Fill the lunchbox to capacity with water, for example,



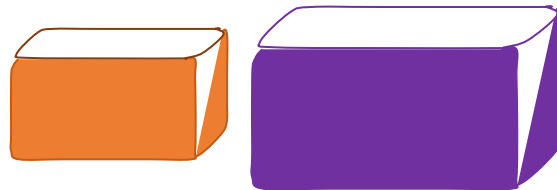
Pour the water from the lunchbox into the measuring device, for example,



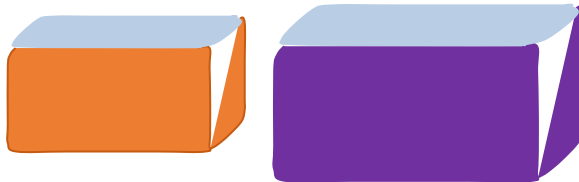
Identify the volume of water.

Record the capacity of the lunchbox in yogurt containers, for example, Capacity = 4 yogurt containers of water

Display 2 more containers, for example,



Fill the containers to capacity with water, for example,



- ▶ Could we measure the volume and capacity of the lunchbox using units of water?
- ▶ How could we measure the volume and capacity of the lunchbox using units of water?
- ▶ Could we measure the volume and capacity of the lunchbox by filling it to capacity with water?

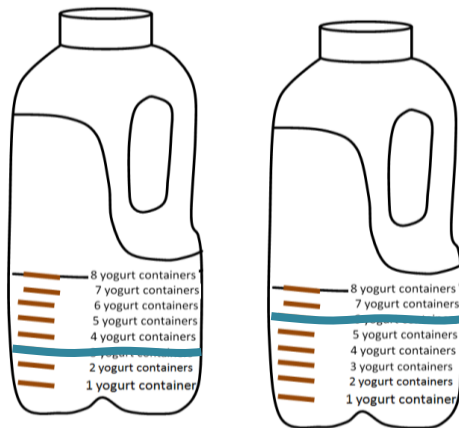
- ▶ Could we then pour the water into the measuring device?

- ▶ Could we identify the volume of water in the measuring device?
- ▶ Is the volume of water in the measuring device, 4 yogurt containers?
- ▶ If the volume of water in the measuring device is 4 yogurt containers, is the capacity of the lunchbox 4 yogurt containers?

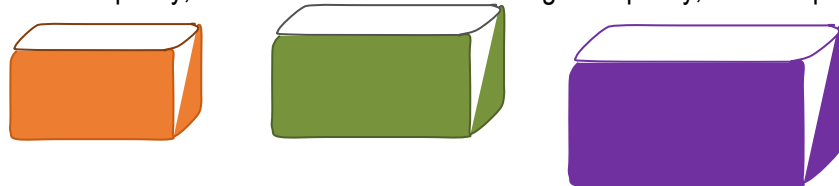
- ▶ Let's measure the capacity of more containers.
- ▶ How could we measure the capacity of these containers?

- ▶ Could we fill the containers with water?

Pour the water from the containers, one at a time, into the measuring device, for example,

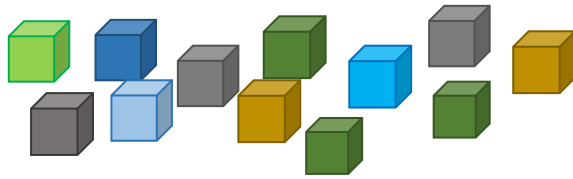


Place the container with the smallest capacity first, then the container with the 2nd smallest capacity, then the container with the largest capacity, for example,

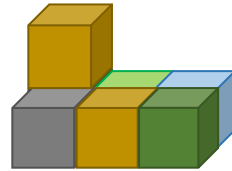


- ▶ Could we pour the water into the measuring device to measure the volume of water?
 - ▶ What is the capacity of this container?
 - ▶ Is the capacity of this container, 3 yogurt containers?
 - ▶ What is the capacity of this container?
 - ▶ Is the capacity of this container, 6 yogurt containers?
-
- ▶ How could we place these containers in order of capacity?
 - ▶ Could we place the container with the smallest capacity first, then the container with the 2nd smallest capacity, then the container with the largest capacity?
-
- ▶ How could we describe the capacity of the containers?
 - ▶ Is the first container's capacity smaller than the second container's capacity?
 - ▶ Is the second container's capacity smaller than the third container's capacity?
 - ▶ So is the first container's capacity also smaller than the third container's capacity?
 - ▶ Is the third container's capacity larger than the second container's capacity?
 - ▶ Is the second container's capacity larger than the first container's capacity?
 - ▶ So is the third container's capacity also larger than the first container's capacity?

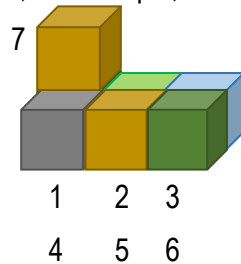
Display some cubes, for example,



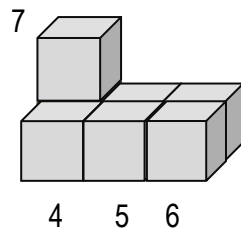
Make a model using some of the cubes, for example,



Count the cubes, for example,



Record the model and the volume, for example,



Volume = 7 cubes

NB: Children's capacity to record the cubes will vary greatly. We're not looking for expertise - we're providing the opportunity for children to develop their capacity to visualise a three-dimensional object in two-dimensional space. Accept and encourage all children's attempts, encouraging them to try to show the three-dimensional aspect. Children could engage in drawing prisms in art lessons prior to and concurrent with this investigation.

► **Today we're going to investigate making models using cubes, then measuring their volume.**

► How could we use these cubes to make a model? Let's investigate!

► How could we measure the volume of this model?

► Could we count the cubes?

► How many cubes did we use to make this model?

► Did we use 7 cubes to make this model?

► What is volume?

► Is volume the amount of space that an object takes up?

► Does this model take up space?

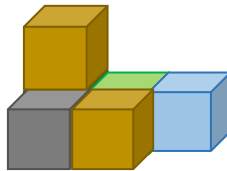
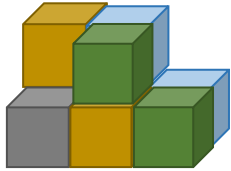
► Could we measure the model's volume?

► Does this model take up the space of 7 cubes?

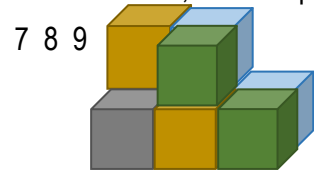
► Is this model's volume, 7 cubes?

► How could we record this?

Make 2 more models using some of the cubes, for example,



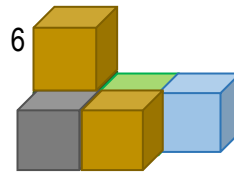
Count the cubes, for example,



7 8 9

1 2 3
4 5 6

Volume = 9 cubes

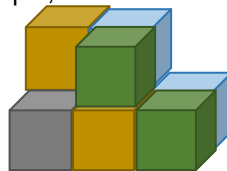
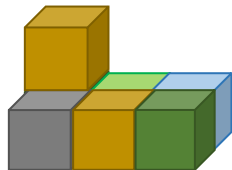
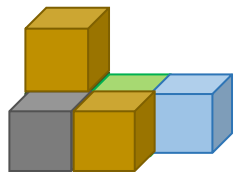


6

1 2 3
4 5

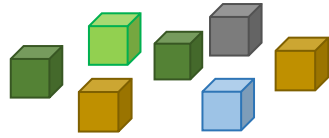
Volume = 6 cubes

Place the model with the smallest volume first, then the model with the 2nd smallest volume, then the model with the largest volume, for example,



- ▶ Let's make more models using cubes.
- ▶ How could we measure the volume of these models?
- ▶ Could we count the cubes?
- ▶ How many cubes did we use to make this model?
- ▶ Did we use 9 cubes to make this model?
- ▶ Is this model's volume, 9 cubes?
- ▶ How many cubes did we use to make this model?
- ▶ Did we use 6 cubes to make this model?
- ▶ Is this model's volume, 6 cubes?
- ▶ How could we place these models in order of volume?
- ▶ Could we place the model with the smallest volume first, then the model with the 2nd smallest volume, then the model with the largest volume?
- ▶ How could we describe the volumes of the models?
- ▶ Is the first model's volume smaller than the second model's volume?
- ▶ Is the second model's volume smaller than the third model's volume?
- ▶ So is the first model's volume also smaller than the third model's volume?
- ▶ Is the third model's volume larger than the second model's volume?
- ▶ Is the second model's volume larger than the first model's volume?
- ▶ So is the third model's volume also larger than the first model's volume?
- ▶ Let's investigate making models with the same volume.

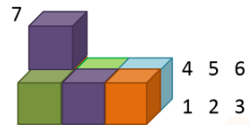
Display 7 cubes, for example,



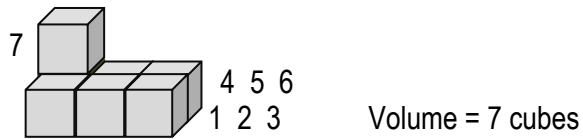
Make a model using the 7 cubes, for example,



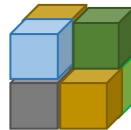
Count the cubes, for example,



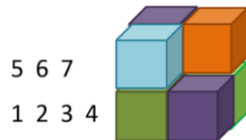
Record the model and the volume, for example,



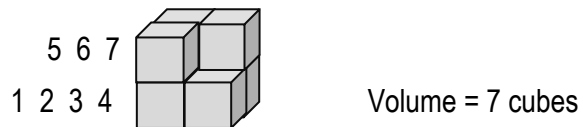
Make a different model using the 7 cubes, for example,



Count the cubes, for example,

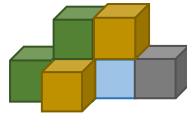


Record the model and the volume, for example,



- ▶ How many cubes?
 - ▶ Are there 7 cubes?
 - ▶ How could we use these 7 cubes to make a model?
 - ▶ How many cubes did we use to make this model?
 - ▶ Did we use 7 cubes?
 - ▶ Could we measure the model's volume?
 - ▶ Does this model take up the space of 7 cubes?
 - ▶ Is this model's volume, 7 cubes?
 - ▶ How could we record this?
-
- ▶ Could we rearrange the cubes to make a different model using the same number of cubes?
-
- ▶ How many cubes did we use to make this model?
 - ▶ Could we measure the model's volume?
 - ▶ Does this model take up the space of 7 cubes?
 - ▶ Is this model's volume, 7 cubes?
 - ▶ How could we record this?
-
- ▶ Could we rearrange the cubes to make another different model using the same number of cubes?

Make a different model using the 7 cubes, for example,

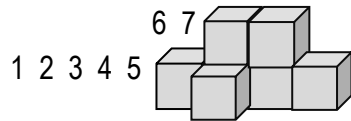


Count the cubes, for example,

6 7
1 2 3 4 5



Record the model and the volume, for example,

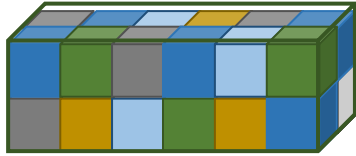


Volume = 7 cubes

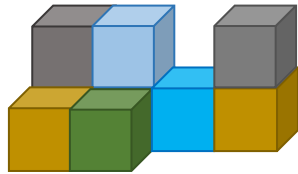
- ▶ How many cubes did we use to make this model?
- ▶ Could we measure the model's volume?
- ▶ Does this model take up the space of 7 cubes?
- ▶ Is this model's volume 7 cubes?
- ▶ How could we record this?
- ▶ Do all of these models have the same volume?
- ▶ Are all of these models the same shape?
- ▶ If we rearrange the cubes, does the volume change or stay the same?
- ▶ Can models of different shapes have the same volume?

- ▶ We've investigated finding the volume and capacity of objects with flat surfaces and straight lines by packing them with cubes.

Display an object packed with cubes, for example,



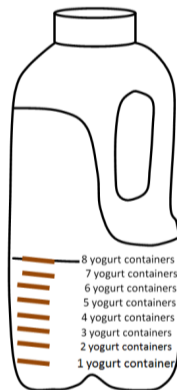
Display a model made from cubes, for example,



Display a container with curved surfaces, for example,



Display the measuring device constructed previously, for example,



► We've investigated finding the volume and capacity of models made from cubes, by counting the number of cubes.

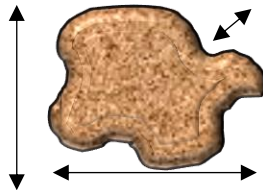
► We've investigated finding the volume of containers with curved surfaces by filling them with water and measuring the amount of water.

► **How could we measure the volume of an object with curved surfaces, if it was solid so we couldn't fill it with water? Let's investigate!**

Display a solid object with curved surfaces that will sink in water, for example, some plasticine



Indicate the object's dimensions, for example,



Display the object, demonstrating it is not hollow.

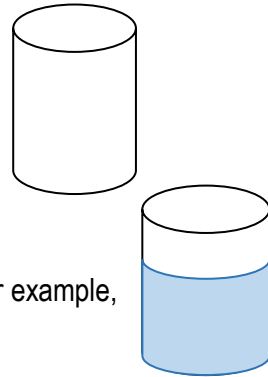


- ▶ What is this object?
- ▶ How could you describe it?
- ▶ How many dimensions does it have?
- ▶ What are the 3 dimensions?
- ▶ Does it have length forwards and backwards? Is that one dimension?
- ▶ Does it have length left and right? Is that another dimension?
- ▶ Does it have length up and down? Is this another dimension?

- ▶ Does this object take up space?
- ▶ Does this object have volume?
- ▶ Because this object takes up space, it does have volume.
- ▶ Does this object have capacity?
- ▶ Because this object is not hollow, it does not have capacity.

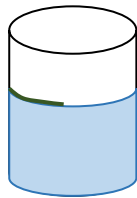
- ▶ How could we measure how much space this object takes up?
- ▶ How could we measure the object's volume?
- ▶ Could we pack this object with cubes?
- ▶ We can't pack the object with cubes because it is not hollow.
- ▶ Could we make this object out of cubes?
- ▶ We can't make this object out of cubes because it has curved surfaces.
- ▶ Could we fill this object with water?
- ▶ We can't fill this object with water because it is not hollow.
- ▶ If we can't measure its volume by packing it or making it with cubes, or by filling it with water, how could we measure its volume?
- ▶ We'd have to use water because the object has curved surfaces.
- ▶ But we can't fill it to capacity with water.

Display a transparent container, for example,



Partially fill the transparent container with water, for example,

Make a mark on the outside of the transparent container with a marker to show the level of water, for example,



Display the solid object, for example,

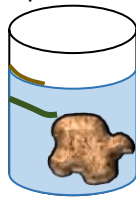


Place the solid object into the container of water, for example,

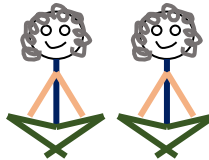


- ▶ So if we can't place water into it, could we place it in water? Let's investigate!
- ▶ Here we have a container.
- ▶ Let's pour in some water.
- ▶ Where does the water come up to?
- ▶ How could we make a mark on the container to show the level of the water?
- ▶ What do you think would happen to the level of the water, if we placed this object into this container of water? Let's investigate!
- ▶ What happened to the level of the water?
- ▶ Did the water level go up?
- ▶ Why?
- ▶ Did the object take up some space in the water?
- ▶ Where does the water come up to now that we have placed the object into it?
- ▶ How could we make a mark on the container to show the level of the water with the object in it?

Make a mark on the outside of the transparent container with a marker to show the level of water with the object in it, for example,



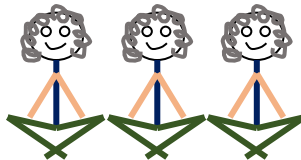
Have two children sit side by side with a small space between them, for example,



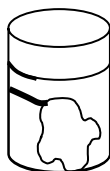
Have another child attempt to sit in the small space between them, for example,



Have one child move over to give the new child space to sit, for example,



Display the transparent container with the object in it, marked with the 2 water level marks, for example,



Record, for example,

The object displaced some water.

- ▶ Why did the water level rise? Let's see if we can investigate!
- ▶ Have you ever been sitting on the floor next to a friend?
- ▶ Are you taking up space?
- ▶ Do you have volume?
- ▶ Has someone else ever come and sat down between you?
- ▶ Will the other child take up space?
- ▶ Will the other child have volume?
- ▶ Is there enough space for the other child to sit between you?
- ▶ Will one of you need to move over to allow the child to fit in the space between you?
- ▶ Is this child now taking up the space that you were taking up before?
- ▶ Did this child displace you?
- ▶ Did the object take up some of the space that the water was taking up before?
- ▶ Did the object displace some of the water?
- ▶ How could we record this?
- ▶ What do you think will happen to the level of the water, if we place each of these objects into this container of water?
- ▶ Do you think the water level will go up?
- ▶ Will the objects displace some water?

Display a transparent container of water with the water level marked, and 2 solid objects of a different volumes that will sink, for example, 2 pieces of plasticine, or a stone and a rock, for example



- ▶ Which object do you think will displace more water?
- ▶ Do you think the object with the larger volume will displace more water?
- ▶ Which object do you think will displace less water?
- ▶ Do you think the object with the smaller volume will displace less water?