

MULTIPLICATIVE RELATIONSHIPS BETWEEN FRACTIONS.

INVESTIGATIONS OVERVIEW PAGE

THIS PAGE IS A SUMMARY OF THE INVESTIGATIONS THAT STUDENTS MAY ENGAGE IN TO DEEPEN THEIR RELATIONAL UNDERSTANDING. INVESTIGATIONS WITH INSTRUCTIONS TO STUDENTS FOLLOW ON SUBSEQUENT PAGES.

- Children investigate different ways that they can create fractions, developing their understanding of the multiplicative relationships between fractions.
 - ▶ creating **halves** by halving 1, explaining a half is called a half because it is half of 1
 - ▶ creating **quarters** by **folding a half, then in half again**, explaining that a quarter is called a quarter because it is a quarter of 1, and a quarter is half of a half
 - ▶ creating **eighths** by **folding in quarters then in half**, and by **folding in halves then in quarters**, explaining that an eighth is called an eighth because it is an eighth of 1, and an eighth is half of a quarter, and an eighth is quarter of a half **Reflection: How can we describe multiplicative relationships between fractions?**
 - ▶ creating **fifths** by fifthing 1, explaining a fifth is called a fifth because it is a fifth of 1
 - ▶ creating **tenths** by **folding in fifths and then halving**, and by **folding in halves and then fifthing**, explaining that a tenth is half of a fifth, and that a tenth is fifth of a half. **Reflection: How can we describe multiplicative relationships between fractions?**
 - ▶ creating thirds by thirding 1, explaining a third is called a third because it is a third of 1
 - ▶ creating sixths by **folding in thirds then halving**, and by **folding in halves and then thirding**, explaining that a sixth is called a sixth because it is a sixth of 1, and a sixth is half of a third, and a sixth is a third of a half **Reflection: How can we describe multiplicative relationships between fractions?**
 - ▶ creating twelfths by **folding in sixths then halving**, by **folding in halves and then sixthing**, by **folding in thirds and then quartering**, and by **folding in quarters and then thirding**, explaining that a twelfth is called a twelfth because it is a twelfth of 1, a twelfth is half of a sixth, and a twelfth is sixth of a half, a twelfth is quarter of a third, and a twelfth is third of a quarter. **Reflection: How can we describe multiplicative relationships between fractions?**

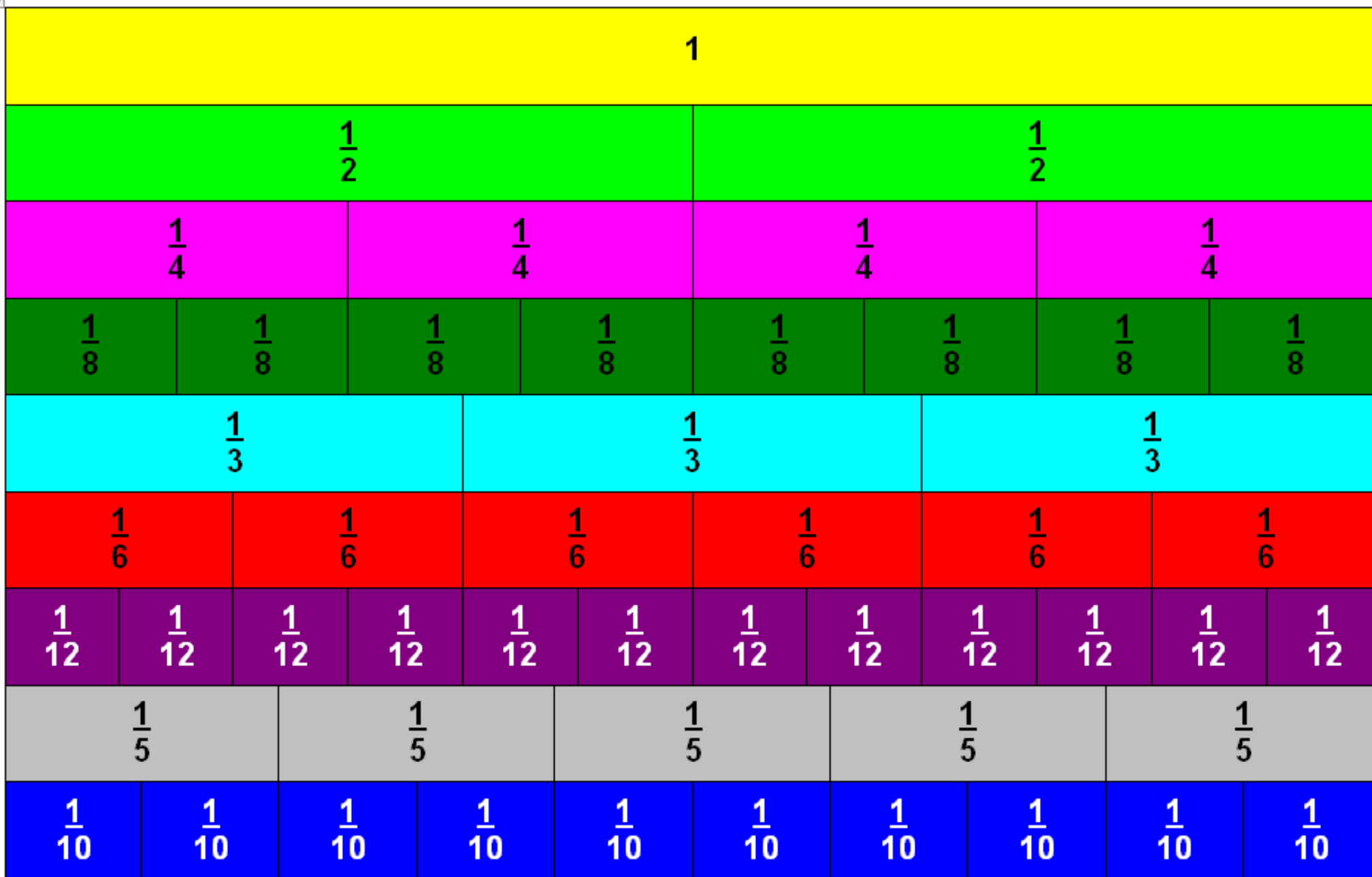
NB: The learning intention is to develop understanding of the importance of creating fractions from the same-sized whole so that we can compare the relative sizes of the fractions, both to one and to other fractions. The learning is in the strategies children use to create fraction, using relationships to other fractions (halving a half to make a quarter, therefore a quarter is half of a half) and estimating fractions that cannot be created through halving (thirds and fifths). Explaining their strategies and the relationships they have discovered will deepen understanding. Creating absolutely perfectly sized fractions is not important. As the learning involved is vast, the investigations may be spread over many lessons. Children could also investigate sevenths and ninths!

- Once children have made their fraction wall, they will not be using it to investigate fractions as it will not be accurate. They will then move on to using a commercially available fraction wall, (or print cut up the attached [fraction wall](#)) placing fractions on the wall that have multiplicative relationships to a fraction already on the wall. **Reflection: How can we describe multiplicative relationships between fractions?**

- In pairs, children have identical shapes of paper, for example, squares or circles. They divide the shapes into halves, quarters, eighths, thirds, sixths, twelfths, fifths and tenths. They explain how they have created each fraction, when possible, in more than one way. They explain the multiplicative relationships between the fractions.

Reflection: How can we describe multiplicative relationships between fractions?

When dividing by 2, 4, 3 and 5 in Year 3, and by 9, 6, 8 and 7 in Year 4, children record their number sentence as a division and as a fraction, for example, $24 \div 4 = 6$ and $\frac{1}{4}$ of $24 = 6$ (see Multiplication and Division by 2, 4, 3, 5, 9, 6, 8, and 7)

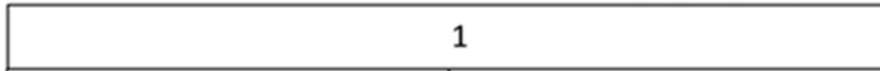


Multiplicative Relationships between Fractions

Halves, Quarters and Eighths

Create equal strips of paper by ruling rows 1 ruler width.

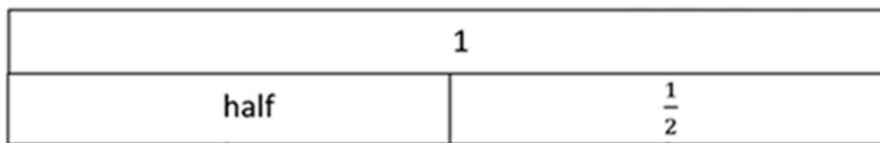
Keep 1 strip whole and place it at the top of the fraction wall as 1.



Divide one strip into halves by dividing into 2 equal parts.

Place the halves under the 1 to continue the fraction wall.

What fraction of 1 is a half?

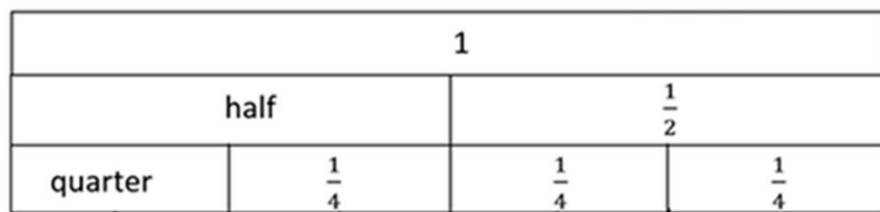


Divide one strip into quarters by halving a half.

Place the quarters under the halves to continue the fraction wall.

What fraction of 1 is a quarter?

What fraction of a half is a quarter?



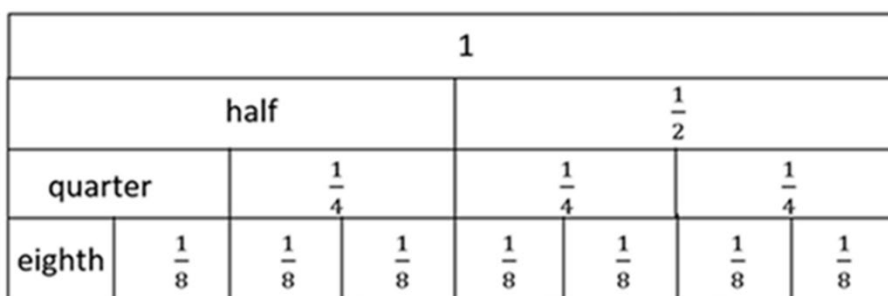
Divide one strip into eighths by halving a quarter.

Place the eighths under the quarters to continue the fraction wall.

What fraction of 1 is an eighth?

What fraction of a quarter is an eighth?

What fraction of a half is an eighth?



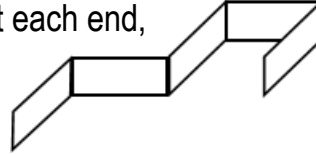
Reflection: How can we describe multiplicative relationships between fractions?

Multiplicative Relationships between Fractions

Fifths and Tenths

Create equal strips of paper by ruling rows 1 ruler width.

Divide one strip into fifths by estimating a fifth at each end, then estimating the remaining 3 fifths.



Place the fifths under the eighths to continue the fraction wall.

What fraction of 1 is a fifth?

| | | | | | | | |
|---------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1 | | | | | | | |
| half | | | | $\frac{1}{2}$ | | | |
| quarter | $\frac{1}{4}$ | | $\frac{1}{4}$ | | $\frac{1}{4}$ | | |
| eighth | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| fifth | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | |

Divide one strip into tenths by halving a fifth, or by fifthing a half.

Place the tenths under the fifths to continue the fraction wall.

What fraction of 1 is a tenth?

What fraction of a fifth is a tenth?

What fraction of a half is a tenth?

| | | | | | | | | | |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1 | | | | | | | | | |
| half | | | | | $\frac{1}{2}$ | | | | |
| quarter | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | |
| eighth | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| fifth | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | |
| tenth | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |

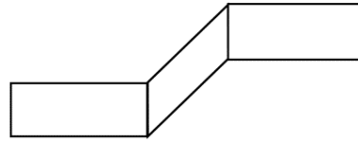
Reflection: How can we describe multiplicative relationships between fractions?

Multiplicative Relationships between Fractions

Third and Sixths

Create equal strips of paper by ruling rows 1 ruler width.

Divide one strip into thirds by estimating a third, then estimating the remaining 2 thirds.



Place the thirds under the tenths to continue the fraction wall.

What fraction of 1 is a third?

| | | | | | | | | | |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1 | | | | | | | | | |
| half | | | | | $\frac{1}{2}$ | | | | |
| quarter | | $\frac{1}{4}$ | | $\frac{1}{4}$ | | $\frac{1}{4}$ | | $\frac{1}{4}$ | |
| eighth | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| fifth | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | |
| tenth | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |
| third | | | $\frac{1}{3}$ | | | $\frac{1}{3}$ | | | |

Divide one strip into sixths by halving a third, or by thirding a half.

Place the sixths under the thirds to continue the fraction wall.

What fraction of 1 is a sixth?

What fraction of a third is a sixth?

What fraction of a half is a sixth?

| | | | | | | | | | |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1 | | | | | | | | | |
| half | | | | | $\frac{1}{2}$ | | | | |
| quarter | | $\frac{1}{4}$ | | $\frac{1}{4}$ | | $\frac{1}{4}$ | | $\frac{1}{4}$ | |
| eighth | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| fifth | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | |
| tenth | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |
| third | | | $\frac{1}{3}$ | | | $\frac{1}{3}$ | | | |
| sixth | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

Reflection: How can we describe multiplicative relationships between fractions?

Multiplicative Relationships between Fractions

Twelfths, Sixths, Halves, Thirds and Quarters

Create equal strips of paper by ruling rows 1 ruler width.

Divide one strip into twelfths by halving a sixth and by sixthing a half and by quartering a third, and by thirding a quarter.

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|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1 | | | | | | | | | | | |
| half | | | | | | $\frac{1}{2}$ | | | | | |
| quarter | | | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | |
| | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |
| fifth | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | |
| tenth | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ |
| third | | | | $\frac{1}{3}$ | | | | $\frac{1}{3}$ | | | |
| sixth | | $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ | | $\frac{1}{6}$ | |
| twelfth | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ |

Place the twelfths under the sixths to continue the fraction wall.

What fraction of 1 is a twelfth?

What fraction of a sixth is a twelfth?

What fraction of a half is a twelfth?

What fraction of a third is a twelfth?

What fraction of a quarter is a twelfth?

Reflection: How can we describe multiplicative relationships between fractions?

Multiplicative Relationships between Fractions

Once you have constructed your own fraction wall, explaining multiplicative relationships between the fractions as you build it, you may use a commercially available fraction wall to continue to investigate the multiplicative relationships.

Build the fraction wall by:

- Starting with 1, identifying that all of the fractions are fractions of this sized 1.
- Placing a fraction on the wall that has a multiplicative relationship to the fractions already on the fraction wall.

For example, you may select any fraction as your first fraction because every fraction has a multiplicative relationship to 1.

Let's select thirds.

Explain the multiplicative relationships on the fraction wall, for example a third is a third of 1.

Our next fraction must have a multiplicative relationship to thirds. Let's select sixths.

Explain the multiplicative relationships on the fraction wall, for example, a sixth is a sixth of 1, a sixth is half of a third.

Our next fraction must have a multiplicative relationship to thirds and / or sixths. Let's select twelfths.

Explain the multiplicative relationships on the fraction wall, for example, a twelfth is a twelfth of 1, a twelfth is half of a sixth, a twelfth is quarter of a third.

Continue building the fraction wall placing fractions on the wall that have a multiplicative relationship to fraction/s already on the wall, explaining all multiplicative relationships.

Reflection: How can we describe multiplicative relationships between fractions?

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|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|--|
| 1 | | | | | | | | | | | | |
| third | $\frac{1}{3}$ | | | | $\frac{1}{3}$ | | | | | | | |
| sixth | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | |
| twelfth | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | $\frac{1}{12}$ | |
| quarter | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | | $\frac{1}{4}$ | | | | | |
| eighth | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ | |
| half | | | | | | $\frac{1}{2}$ | | | | | | |
| tenth | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | $\frac{1}{10}$ | |
| fifth | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | $\frac{1}{5}$ | | | | | |

Multiplicative Relationships between Fractions

Have identical shapes of paper, for example, squares or circles.

Divide the shapes into halves, quarters, eighths, thirds, sixths, twelfths, fifths and tenths.

Explain how you have created each fraction.

Explain the multiplicative relationships between the fractions.

Reflection: How can we describe multiplicative relationships between fractions?