Children's Understanding of Two-Digit Place Value: A Place for Place Value in Pre-k Mathematics Instruction

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Abstract

The development of young children's number sense and understanding of the base 10 system is essential for the acquisition of more complex number skills in later years. Although there has been an increased effort to explore the development of these concepts in early elementary grades (K-5), research in pre-kindergarten (pre-k) has been limited. This paper begins by presenting a theoretical framework, drawing on the definition of mathematics understanding developed Carpenter & Lehrer (1999), suggesting that the most basic level of two-digit place value understanding can be achieved in pre-k. Next, we describe a series of five pre-k mathematics activities, a subset of the larger *MyTeachingPartner Math-Science* (*MTP-MS*) curriculum, and describe how these specific lessons employ color-coded ten-frame representations to scaffold children's two-digit place value understanding. Finally, we present the results of a pilot administration of a dynamic *Number Sense and Place Value Assessment* to 44 randomly selected students across six classrooms participating in the *MTP-MS* project. Our results suggest that in two-digit numeral representations, children's understanding of the ones place develops before knowledge of the tens place. The findings of the study are discussed in relation to pre-k mathematics contexts, with implications for early childhood mathematics instruction.

Children's Understanding of Two-Digit Place Value: A Place for Place Value in Pre-K Mathematics Instruction

Most mathematics educators and experts are in agreement that students' understanding of the principles underlying place value is critically important (Baroody, 1990; Cotter, 2000; Hiebert & Wearne, 1992; Fuson & Briars, 1990; Ho & Cheng, 1997; Miura, Okamota, Kim, Steere, & Fayol, 1993; Resnick, 1983; Ross, 1986; Thompson & Bramald, 2002; Van de Walle, 2003), but they often disagree about how and at what age such concepts should be instructionally targeted. For instance, Fuson and Briars (1990) argue that place value instruction should be delayed until at least first or second grade and taught in concert with multi-digit addition and subtraction problems. Likewise, Ross (1986) proposes a five-stage model of place value development that begins in second grade and concludes in fifth grade, requiring students to possess an understanding of numerical part-to-whole relationships before entering stage one of the model. This position tends to be that of most of the early childhood mathematics community: Students should possess knowledge of multidigit addition and subtraction or an understanding of part-to-whole number relationships prior to place value instruction. Consequently, exposure to place value concepts is typically reserved for grades two and up, and is rarely targeted in pre-k mathematics instruction. However, a possible unfortunate consequence of this delayed exposure is student misunderstanding of place value, documented in extensive literature on early and middle elementary mathematics learning (Carpenter & Moser, 1984; Cobb & Wheatley, 1988; Hiebert & Wearne, 1992) particularly in multidigit addition and subtraction (Fuson, 1990b). These misunderstandings may persist until fifth or sixth grade for some students (Kamii, 1986).

In contrast to the conventional views that place value instruction should be reserved for second grade and above (Fuson & Briars, 1990; Ross, 1986) a few researchers hold an "early exposure" philosophy. For example, Baroody (1990) argues that exposure to foundational place value concepts (e.g., exposure to multiunit meanings, working with two-digit numbers) should be introduced much earlier than first or second grade, and can begin as soon as children begin working with two-digit numbers. Baroody's rationale for early exposure rests on the assertion that young children have the ability to make basic connections and establish foundational understanding for later mathematics. As Baroody (1990) suggests, "By introducing multiunit meanings concretely as soon as children begin using two-digit numbers in school and discussing them throughout the primary grades, children may develop a more secure basis for understanding multiunit concepts" (p. 282). Similarly, Irons (2002) promotes an early exposure philosophy, suggesting that, "it is not necessary to establish all of the intricacies of place value before children work with two-digit or larger numbers. Children can use many ideas from other representations to assist them as they work with larger numbers" (p. 4). In other words, the philosophies held by Baroody and Irons suggest that the foundational knowledge for understanding of two-digit place value can be targeted well before children engage in multidigit addition and subtraction; instructional activities which typically occur in elementary grades two to five.

We have adopted a position similar to that held by Baroody (1990) and Irons (2002). In this paper, we demonstrate ways that instruction related to place value can be developmentally appropriate for pre-k, if it is focused at the most foundational level. We are not suggesting that higher level place value concepts (i.e., understanding of computational algorithms or multidigit addition and subtraction operations, scientific notation, or decimals) should be taught in pre-k mathematics. Instead, our recommendations for place value instruction emphasize a basic understanding of the component parts of two-digit numerals (tens vs. ones). We demonstrate that place value can have a place in pre-k mathematics instruction by offering:

1) A theoretical framework for pre-k children's development of place value understanding that can occur before any exposure to multidigit addition and subtraction instruction
2) Examples of place value activities developed by experts in early childhood mathematics education and delivered in seven classrooms as part of a comprehensive pre-k mathematics curricula
3) Descriptive statistics from a pilot administration of our *Number Sense and Place Value Assessment*, which indicates that young children can have foundational understandings of two-digit place value.

Theoretical Framework

To our knowledge, no previous research inquiries have specifically investigated if and how two-digit place value understanding emerges in pre-k. Ross (1986) proposed a comprehensive five-stage model of place development for early elementary children; however this model begins with second grade and concludes in fifth grade. Due to the lack of research surrounding place value development in pre-k, we summarize Ross's five-stage model to provide context and describe how two-digit place value understanding is thought to emerge in later grades. We also evaluate the stages of this model to determine the number sense concepts that may be prerequisite to this model, to help explore understandings that can be targeted in pre-k.

When they are in stage one of the model, Ross (1986) suggests that children can read and write two-digit numerals and associate the whole numeral with the quantity it represents. However at this stage children assign no meaning to the individual parts which comprise the two digit numeral. In other words, a child can read and write "37" and understands that "37" represents 37 objects, but neither the 3 nor the 7 carry any meaning except parts of the whole numeral. Ross's research suggests that most children reach stage one by the beginning of the second grade. Children in stage two begin to interpret the digit on the right as representing the "ones place" and the digit on the left indicating the "tens place." However, this understanding does not extend to the quantities indicated by each part. The model indicates that the majority of children reach stage two by the fall of second grade.

In stage three of Ross' model, children are able to interpret each digit as representing the number indicated by its face value (i.e., tens vs. ones). For example, a child may verbally label the numeral "4" in the number 48 as the "tens" but in this stage the child does not recognize that the number represented by the tens digit is a multiple of ten. Ross suggests that the majority of students pass through stage three during fourth grade. A child in stage four knows that the left digit in a two-digit numeral represents sets of ten objects and the right digit represents the remaining objects; however, this knowledge is still tentative and task performance is unreliable. By the end of the fourth grade, Ross contends, most students have reached stage four. In the fifth and final stage, children know that the individual digits in a two-digit numeral represent different parts (tens part vs. ones part). Most children achieve this stage after completing the fifth grade, understanding the part-whole relationships of two-digit numerals.

While Ross's model of two-digit place value begins with the second grade, a growing body of research suggests that many young children possess a surprisingly extensive set of informal mathematics skills (Bryant, 1995; Ginsburg, Klein & Starkey, 1998; Zur & Gelman, 2004), as well as the interest, curiosity, and willingness to explore large numbers (Sarama & Clements, 2007). We assert that children as young as four may have the cognitive ability to understand basic place value related concepts. In this paper, we outline a theoretical framework suggesting a set a prerequisite developmental skills necessary to acquire the most basic two-digit place value understanding. Figure 1 provides a conceptual map of our theoretical framework and the following paragraphs describe the component parts and relationships that exist within the framework.

Mathematics Understanding in Pre-K

Mathematics understanding in pre-k, like that at any other grade, can be achieved at many different levels. For our work in this area, we have adopted Carpenter and Lehrer's (1999) definition of mathematics understanding, one that suggests, "understanding is not an all-or-none phenomenon… virtually all complex ideas or processes can be understood at a number of levels and in quite different ways" (p. 20). In other words, understanding is developed on a continuous scale, rather than dichotomously (as suggested by Ross's [1986] five-stage model for progressive development of two-digit place value understanding). Carpenter and Lehrer's definition of mathematical understanding is particularly relevant to our theoretical framework; It can be presumed that no pre-k students will have had prior exposure to two-digit place value concepts, and perhaps minimal exposure to one-digit number sense concepts, but that all will pass through a continuum of understanding as their conceptual skills emerge. Therefore, in our theoretical framework we categorize students' understanding of all number sense and two-digit place value-related concepts on a continuum that moves from low to high. In this case, a complete understanding of every component included in the theoretical framework is not our expectation; rather we feel that early exposure to foundational concepts will help students to make connections, develop numerical flexibility, and generally support students' basic understanding of two-digit place value.

Coordinating Number Sense Concepts

Basic understanding of place value involves building relationships and making connections between key ideas, for example quantifying sets of objects in groups of ten and treating the groups as single units (Steffe & Cobb, 1988; Fuson, 1990b). Our theoretical framework asserts that this basic understanding begins with the coordination of concrete and abstract number sense concepts. For the purposes of this theoretical framework, we classify concrete number sense skills as those that can be memorized or learned in a rote manner, for example, learning the spoken words for the counting numbers (e.g., *one, two, three, …)* or acquiring numeral recognition skills. Abstract number sense skills, in contrast, include numerical concepts such as one-to-one correspondence, cardinality, a conceptual understanding of zero, and a conceptual understanding of one group of ten objects.

Developing One-Digit Number Sense.

Our theoretical model suggests that the coordination of concrete and abstract number sense skills for one-digit numbers serves as a foundation for two-digit place value understanding—as students memorize the counting numbers (1-9) and learn to recognize numerals (0-9), they are essentially learning the prerequisite skills required to understand the meaning of the ones place in a two-digit numeral. Several pre-k mathematics curricula, including *Building Blocks* (Clements & Sarama, 2006; Sarama, 2004; Sarama & Clements, 2003, 2004), *Numbers Plus* (Epstein, 2003), and *Head Start Math* (Sophian, 2004) target the development of concrete and abstract number sense concepts (e.g., cardinality and one-to-one correspondence). *Big Math for Little Kids* (Ginsburg, Greens, & Balfanz, 2003; Greenes, Ginsburg, & Balfanz, 2004; Morgenlander & Manlapig, 2006) extends this knowledge to two-digit numbers.

Developing Two-Digit Number Sense.

We hypothesize that instruction involving two-digit numerals is extremely limited in pre-k, most likely due to the belief that exposure to positional place value concepts in two digit numbers is developmentally inappropriate or too cognitively demanding. With our theoretical framework we suggest that a basic understanding of two-digit place value is possible at younger ages, and is supported when children are able to overcome two important conceptual hurdles, namely, the conceptual understanding of zero and the understanding of a “group of ten”. Although most pre-k mathematics instruction does not focus on zero, we include it in our theoretical framework because we feel a conceptual understanding of zero directly impacts children’s acquisition of two-digit place value understanding. Our primary rationale for targeting zero rests on the assumption that the number ten is the first two-digit numeral introduced to students during formal mathematics instruction and therefore an understanding of zero will theoretically increase the likelihood that one can understand the relationships between the component parts of two-digit numbers (e.g., 10 = 1 group of 10 and 0 ones). Further, a lack of conceptual understanding of zero could potentially be a developmental barrier to children’s place value understanding in later grades (Baroody, Gannon, Berent, & Ginsburg, 1983) as zeros, particularly embedded zeros in three-digit numbers or larger (e.g., 105 or 4029) are ignored due to children’s misconception that zero is insignificant and does not represent a placeholder in the base ten notation (Kamii, 1981). Therefore, in our framework, understanding of zero serves as an important conceptual link to help students make connections between one- and two-digit numbers, and may also influence later place value understanding.

The coordination of all of the number sense skills, with a particular emphasis on children's conceptual understanding of zero and making groups of ten, will help to achieve the most basic two-digit place value understanding. Once students have mastered one-digit number sense, and have developed conceptual understandings for zero and groups of ten, then they can advance to two-digit numbers, repeating the sequence of coordinating numeration knowledge and number concepts with these larger, two-digit numbers. A primary caveat with two-digit numbers, however, is that now the independent parts of two-digit numbers take on positional meaning. Once armed with this conceptual knowledge, children can coordinate this understanding with their previously acquired knowledge of one-digit numbers, to perform basic two-digit place value specific tasks (e.g., matching ten-frame representations to written numerals).

Scaffolding Foundational Place Value Understandings with Pre-K Activities

MTP Math Curricular Overview

MyTeachingPartner Math-Science (MTP-MS) specifically targets the teaching and learning of children at risk of early school failure, a population for whom achievement gaps in mathematics and science are visible as early as pre-k (NRC, 2009). While the project targets both math and science, we only consider portions of *MTP Math* here. MTP *Math* provides teachers with a set of inquiry-based activities intended to expose children to relevant mathematics concepts and skills, and offers children opportunities to observe, predict, collect, analyze, and communicate their processes and results (Kinzie, Pianta, Kilday, McGuire, & Pinkham, 2009). The curriculum is guided by a comprehensive mathematics trajectory, adapted from the National Council of Teachers of Mathematics (2000) focal areas, the pre-k to grade two trajectories developed by Clements (2004), and further refined based on a review of state pre-k mathematics standards. To provide authentic points of inquiry, our year-long trajectories reflect seasonal changes and are specifically designed to help extend and revisit children's thinking across the year. We emphasize a balanced integration of student-centered, meaningful interactions with teacher-directed, scaffolded target exposures to key mathematics concepts. As Ginsburg and colleagues have done in their research (Ginburg & Goldbeck, 2004), the curriculum instructs teachers to provide opportunities to encourage children's thinking and model/elicit mathematics language to express that thinking. In each *MTP Math* activity children are encouraged to Engage, Investigate, Discuss, and Extend, a modification of the 5E Model (Bybee et al., 2006). We use children's literature to anchor some activities, providing comfortable entry points, further supporting the development of mathematics embedded in story-based, language and literacy contexts (Casey, 2003; Casey, Kersh, & Young, 2004).

MTP Math consists of a total of 57 activities designed to be delivered across the academic year in four specific learning domains: (1) Number, (2) Geometry (3) Operations, and (4) Measurement. Teachers implement two mathematics activities per week, with the total number of activities for the given months as follows: September (7), October (8), November (6), December (6), January (8), February (8), March (8), and April (6). Mathematics activities typically range from 10-25 minutes in length. Approximately half (53%) of the activities are facilitated in whole group settings (entire class) while the remaining 47% of activities are conducted in a small group format (six to eight students). Thirty percent of *MTP Math* activities involve at least a partial reading of a related book or build on the story context of a previously read book. In addition to the full complement of math activities, *MTP Math* supplements selected activities with center-based activities. Teachers employ center cards, double-sided, visually-based cards presenting activities designed to engage students in follow-up discussions, make authentic connections, and explore critical mathematics themes with the teacher in a smaller group setting (ideally one to four students).

MTP Math Number Sense and Place Value Components

Number Sense Focus

Although the domains of *MTP Math* activities include Number, Geometry, Measurement, and Operations, the majority of activities (30 of 57; 53%) include objectives related to number sense. Despite varying and evolving definitions of "number sense" (Berch, 2005) for the purposes of this paper and in the *MTP Math* curriculum we refer to the work of Siegler (1991) and Jordan, Kaplan, Olah, and Locuniak (2006) to define number sense as mathematics understanding that includes identifying written numbers, performing counting activities, organizing numbers in sequence, and making decisions about magnitudes (i.e., comparisons between quantities). Our rationale for the disproportionate focus on number sense is twofold. First, experts collectively agree that the development of early number sense equips students with important foundational and prerequisite skills that prepare students to learn more complex mathematics concepts such as place value, number composition and decomposition, basic arithmetic operations, and understanding of mathematical properties (Baroody, 2009; Griffin, 2004; Jordan, 2007; NCTM 2008; NMAP, 2008; Miura, et al., 1993; Van de Walle, 2003). Second, research suggests that number sense skills acquired in pre-k are not only foundational, but also highly correlated with academic success in early elementary grades (see Jordan, Kaplan, Locuniak, & Ramineni, 2007; Stock, Deoete, & Roeyers, 2009) and can even predict mathematics achievement as late as high school (Duncan et al., 2007; Ginsburg & Allardice, 1984).

MTP Math Number Chart.

Across the year, the *MTP Math Number Chart* (see Figure 2) serves as a pictorial manipulative and instructional anchor, gradually building upon, while simultaneously reviewing and enhancing young children's ability to visualize number representations through appropriate groupings (Cotter, 2000). We believe that repeated exposure to the *MTP Math Number* Chart provides students with opportunities to revisit related one-digit number sense and two-digit place value concepts in a highly visual format. Our instructional methods are based on the hypothesis that building connections between external representations can promote the construction of internal understanding (Hiebert & Wearne, 1992) and therefore we supplement all place value activities with the *MTP Math Number Chart*. The arrangement of numbers in this chart was informed by the number chart employed in *Big Math for Little Kids* (BMLK, Ginsburg, Greenes, & Balfanz, 2003) in which 0 and related numerals (10, 20, and 30), occupy the left-most column in the table; each row ends with the numbers related to 9 (9, 19, 29, 39). To this we have added visual ten-frame representations of the quantity reflected by each numeral, color coded (blue for the tens place, orange for the ones place) to scaffold children's understanding.

We believe the color-coded ten-frame representations displayed in the *MTP Math Number Chart* for both one-digit and two-digit numerals serve as concrete visual analogs, helping students to reduce the processing load of a more complicated concept such as place value (Boulton-Lewis & Halford, 1992). In other words, the color-coded ten-frame representations visually display numbers in a part-to-whole relationship which may help reduce the adjustments children must make when first learning about place value (Fischer, 1990) and assist children in learning to interpret numbers in terms of a tens part and a ones part, an important developmental step in children's place value acquisition (Hunting, 2003; Resnick, 1983). For example, using our color-coded ten-frame representations, each blue ten frame represents a "part" and the orange blocks (ones) leftover would be an additional part, so the number 15 would have two parts (10 and 5), 25 would have three parts (10, 10, and 5) and 35 would have 4 parts (10, 10, 10, and 5).

We also believe that color-coded ten-frame representations help students to construct mental models of numbers (Boulton-Lewis, 1998) and make connections between the three different forms of numerical representations (Barr, 1978). For example, as Hiebert and Wearne (1992) suggest, activities scaffolded with visual representations support students as they "build relationships between quantities and …[act] on quantities that are represented physically, pictorially, verbally, and symbolically." Further, these researchers suggest that visual representations can become "tools that can record quantities, reveal the effects of acting on quantities, and aid in the communicating about these activities with others in the group" (Hiebert & Wearne, p. 99). In other words using the color-coded ten frame representations and corresponding numerals included in the number chart help students to connect: (1) verbal representations (e.g., the oral number name for the word "fifteen"), (2) material representations (e.g., base representation demonstrated by ten-frames), and (3) Symbolic (e.g., written two-digit Arabic numeral "15"). [See Figure 3]. We believe that the color-coded representations are particular helpful in scaffolding children's ability to make connections between the material representation and symbolic because the color-coded digits in the Arabic numerals correspond to the ten-frame representations. Further, the chart serves as an anchor to help children translate between multiple representations, which increases the likelihood that children will accept the equivalence of two different orderings found in place value representations (Ainsworth, 1992).

Finally, the completed *MTP Math Number Chart* makes it easier for children to identify numerical patterns, for example the numbers increase by one unit (one orange block) across each row from left to right, and the columns increase by 10 units (one blue ten-frame) when down the columns.

MTP Place Value Activities.

Of the 30 *MTP Math* activities focusing on number sense, five include objectives related to two-digit place value. These activities (see Appendix A) were designed to support the evolution of children's understandings in the following manner: Beginning in September, we introduce students to single ten-frame representations (see row one of Figure 1) and related manipulatives (e.g., snap cubes), to investigate and build the quantities associated with one-digit numerals, starting with zero. In other words, students begin by building the first row of the *MTP Math Number Chart* (Figure 2) and learn to coordinate their use of number skills such as one-to-one correspondence, cardinality, and numeral recognition. In late January, students are introduced to the two-digit numeral, *10*, and begin working with separate ten-frame representations (See row two of Figure 2). After the pictorial ten-frame representation is introduced to students, the ten-frames are used in subsequent lessons, along with modeling of appropriate mathematical language such as "groups of ten" and "leftovers," to explain the underlying number concepts of two-digit place value. (We avoid terms such as place value, tens place, or ones place.) Consistent and repeated implementation of external representations both within and between *MTP Math* lessons affords students opportunities for repeated exposure to and practice with the ten frames and associated numerals, supporting their acquisition of the prerequisite number sense skills outlined in the theoretical framework that we believe are foundational for basic place value understanding, while inherently anchoring instruction around the number ten (Van de Walle, 2003).

Because there is some disagreement about when instructional activities related to place value can first be productively implemented, we felt the need to develop a measure of children's related understanding. We selected a dynamic assessment model to provide more information about what children's basic competencies might be, as described below. However, we did not implement the measure to test the effects of the *MTP Math* curriculum (something we will be doing after Year 3 of the field trial, and on which we will report in the future).

Pilot Administration of Dynamic Assessment: *Number Sense and Place Value*

Development of the Measure

In order to assess pre-k students' knowledge of number sense and two-digit place value, we developed a two-part dynamic assessment, including 21 total items (13 related to number sense and 8 on two-digit place value). We selected a dynamic assessment model because, unlike conventional assessments that generally indicate only a static indication of performance on any given item (Justice & Ezell, 1999), dynamic assessments provide an indication of the level of support needed for performance (Pena, Iglesias, & Lidz, 2001), and so are more suggestive of an individual's Zone of Proximal Development (Vygotsky, 1978). Further, we are confident that the dynamic assessment model aligns with our theoretical framework, better capturing the levels of understanding of each child, as each item contains a sequence of scaffolds and prompts designed to identify under what conditions a child is able to optimally perform. For each item, children respond to series of prompts until the he or she can correctly answer the question or until all prompts are exhausted. All items on the assessment are administered using a scripted protocol and predetermined scoring system (see Figure 4 for an example) to ensure consistency.

Number Sense Component.

The number sense component of the dynamic assessment evaluates two number sense skills: rational counting (4 items; 16 total points) and numeral recognition (9 items; 15 total points). A total of 31 points are possible on the number sense component of the assessment. Dynamic scaffolds supplemented all 13 items on the number sense component of the assessment. See Figure 5 for a sample of the question/scaffold format for a number sense item examining rational counting skills, *How many teddy bear counters are there?* Here the successive scaffolds include prompts to count the bears, to count with a ten-frame, and finally to point and count together with the administrator.

Place Value Component.

The place value component of the dynamic assessment is used to examine students' two-digit place value understandings. The place value component assesses two types of place value understanding: Understanding of tens place (4 items; 15 points) and understanding of ones place (4 items; 15 points). The majority of these items were designed to measure students' concrete understanding of place value, as reflected by students' ability to match numerals to corresponding ten-frame representations (or vice versa, 8 items total). On four of these eight items, response options differed only in the display of the ones place, and were categorized as items testing ones place understanding. The remaining four items differed only in the display of the tens place, and were categorized as items assessing 10's place understanding. Figure 6 presents a sample item to assess ones place understanding; both response options display an identical quantity of filled ten-frames for the tens place; the two response options vary only in the ten-frame displays of the ones place. Following an incorrect response by a student, the administrator offers the first scaffold, providing a display in which the target number and the ten-frames are both color-coded (blue for the tens place and orange for the ones place). A second scaffold involves the administrator pointing and counting the filled ten-frame displays together with the child, to determine the correct response.

Test Participants

For the pilot administration of this measure, we randomly selected 44 pre-k students between the ages of 54 and 66 months, from six classrooms across five school districts in central Virginia. All children were enrolled in pre-k classrooms participating in a pilot test of the *MyTeachingPartner Math* curriculum. All classrooms included 16-18 students. Demographic data from parent or guardian surveys (approximately 75% response rate) revealed that nearly 50% of the students were White/Caucasian, 30% Black/African American, and the remaining 20% were reported as another race or a mix of racial backgrounds. Just over half of the respondents reported earning an annual income of $35,000 or less. From the 16-18 children in each classroom, we randomly selected eight students for participation in the dynamic *Place Value and Number Sense Assessment*. However, due to absences we were only able to assess 44 total students. We eliminated from participation any student for whom the primary language was not English, as well as any students with Individualized Education Plans (IEP's) for anything other than speech. The demographics of the children selected for the assessment (n = 44) were not significantly different from the overall classroom demographics.

Administration Procedures

Forty four students were randomly selected and administered the dynamic *Number Sense and Place Value Assessment*. The test was administered in individual interview format with each assessment taking between 10-20 minutes to complete. All testing was conducted in a separate area removed from the classroom, with the administrator recording student responses on prepared answer sheets. As indicated above, this administration was not intended as a test of the effects of the *MTP Math* curriculum, but rather as a test of the assessment itself, as a measure of what pre-k students know about number sense and two-digit place value.

Descriptive Data Analysis

To begin the data analysis, the overall raw scores from the number sense and place value components of the dynamic assessment were calculated. Using SPSS, we calculated means and standard deviations of raw scores for all students on the number sense and place value components. Additionally, we conducted a t-test to compare the mean scores (number sense component vs. place value component) to test for statistically significant differences and tested for a correlation between the two component scores.

Results of Test Administration

The results of the component section of the *Number Sense and Place Value Assessment* are displayed in Table 1. For the number sense component of the dynamic assessment (see Table 2), students ability to produce the correct number of objects from a given set appeared to generally decrease as the number of items in the set was increased from 3 to 7 to 8 to 14. When considering the percent of correct responses between level 1 (no scaffolding) and level 2 (student prompted to count the items in the set) for the set sizes of 7, 8, and 14, the average increase was

9.1%. Introducing ten-frames to help scaffold the children's counting resulted in an average percent increase of 15.2% for the same items. We did not include the set size of 3 in these results because virtually all students (97.7%) were able to correctly produce a set of 3 items without any scaffolding, and therefore the effects of scaffolding for this item were minimal.

For the place-value-specific components of the dynamic assessment (ones place understanding vs. tens place understanding) a t-test was conducted to compare the mean scores. We found that students scored significantly higher on items assessing knowledge of ones place understanding (M = 8.80 out of 15 [range = 0 to 15], SD = 4.07) than items assessing tens place understanding (M = 6.41, out of 15 [range = 0 to 13], SD = 4.11); t (86) = 2.74, (p = .007), which suggests children are better at recognizing visual representations of two-digit numbers that differ in the ones place than the tens place. This result is consistent with our theoretical framework that suggests children begin to acquire an understanding of the ones place before an understanding of the tens place, most likely because students spend more time working with and reviewing one-digit numeral representations than similar representations of two-digit numerals. Results for students' two-digit place value understanding, with the effects of the various levels of scaffolding, are displayed in Tables 3 and 4. The addition of color-coded scaffolds increased students' correct response rate by an average of 13% for ones digit items and 14.7% for tens digit items. When the color coding alone did not prove sufficient for encouraging correct responses, an additional point-and-count strategy offered modest amounts of benefit, increasing students' correct response rates by 3.8% for ones digit items and 1.6% for tens digit items, respectively.

Our results indicate that students' number sense and overall place value scores were significantly correlated (r = .511; p < .001), demonstrating a relatively strong relationship between the two outcome scores and suggesting that students' understanding of number sense and two-digit place value may develop in concert, thus supporting our proposed theoretical framework.

Discussion

Although it is the general consensus among researchers and mathematics educators that relational understanding of place value is a multifaceted and developed over time (Bialystok, 1992; Fuson, 1990a; Donlan & Gourlay, 1999; Thomas, Mulligan, & Goldin, 2002), we assert that both the foundational and prerequisite skills associated with two-digit place value understanding have a place in pre-k mathematics instruction. We have provided descriptions of how these understandings may be facilitated in a pre-k math curriculum, and also how they may be measured. The results of our test administration of the dynamic *Place Value and Number Sense Assessment* suggest that number sense skills and two-digit place value understanding are present simultaneously in pre-k students. Further our test results suggest the value of color-coded ten-frame representations as instructional scaffolds to support students' connections between two-digit numerals and the quantity they represent: children's ability to match ten-frame representations with the corresponding numeral increased when these scaffolds were provided in the dynamic assessment.

More research needs to be done to explore the extent to which pre-k students can acquire two-digit place value knowledge, and how this understanding can best be facilitated. In our current work we are administering the measure to a larger population in a field trial (approximately 350 students in 35 classrooms), as one of the outcome indicators for the number sense and place value activities described in this paper. As a result, we will be able to report both on the effects of this instruction, as well as determine whether place value concepts are evident along with number sense, as suggested by our test administration. We will also be able to report more on the psychometric characteristics of the measure (e.g., validity and reliability).

An interesting extension of this research would be to expand the ten-frame representations included in the *MTP Math Number Chart* to three-digit numbers with three different colors, to help students in kindergarten or first grade understand the concepts involved in three-digit place value. This would provide more longitudinal evidence about the evolution of students' place value understanding over time, and perhaps indicate the potential of external representations at different grade levels. By unpacking what young children are capable of learning about place value, we aim to identify and support the instructional strategies that can best support practice, and to better prepare teachers to facilitate related, conceptually-based mathematics understanding in pre-k classrooms.

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Table 1

Means and Standard Deviations on the Component Sections of the Number Sense and Place Value Test

Test Component	Mean	Standard Deviation
Number Sense – Rational Counting (Maximum Score =16)	12.36	3.13
Number Sense – Numeral Recognition (Maximum Score = 15)	7.48	3.84
Place Value – Ones Digit Understanding (Maximum Score = 15)	8.80	4.07
Place Value – Tens Digit Understanding (Maximum Score = 15)	6.41	4.11
Overall Raw Score (Maximum = 61 points)	35.05	11.81

n = 44 student

Table 2

Effects of Dynamic Ten-frame Scaffolds: Percent Correct from the Number Sense Test on Relational Counting

	Number of Items in Set to Be Counted by Child			
	3	7	8	14
Level of Scaffolding				
1. No Scaffolding	97.7	63.6	68.2	29.5
2. Student prompted to count items	97.7	77.3	72.7	38.6
3. Student uses ten-frames to count	97.7	86.4	88.6	59.1
4. Student uses ten-frames and counts with assessor	100	95.5	93.2	70.5

Table 3

Effects of Dynamic Ten-frame Scaffolds. Percent Correct from the Place Value Assessment on Items with Responses Differing in the Ones Place with Levels of Scaffolding Indicated

	Two Digit Numeral Assessed				
	12	13	15	26	Average
Level of Scaffolding					
1. No Scaffolding	50.0	65.2	47.8	60.9	56.0
2. Color-coding of tens and ones place	58.7	82.6	67.4	67.4	69.0
3. Color-coding of tens and ones place, plus point and count with administrator	67.4	82.6	71.7	69.6	72.8

Table 4

Effects of Dynamic Ten-frame Scaffolds. Percent Correct from the Place Value Assessment on Items with Responses Differing in the Tens Place with Levels of Scaffolding Indicated

	Two Digit Numeral Assessed				
	11	18	22	23	Average
Level of Scaffolding					
No Scaffolding	47.8	32.6	32.6	43.4	39.1
Color-coding of tens and ones place	63.0	52.2	43.5	56.5	53.8
Color-coding of tens and ones place, plus point and count with administrator	67.4	54.3	43.5	56.5	55.4

Figure 1

Conceptual Map of Number Sense Skills Leading to Two-Digit Place Value Understanding With Levels of Understanding Developing from Low to High

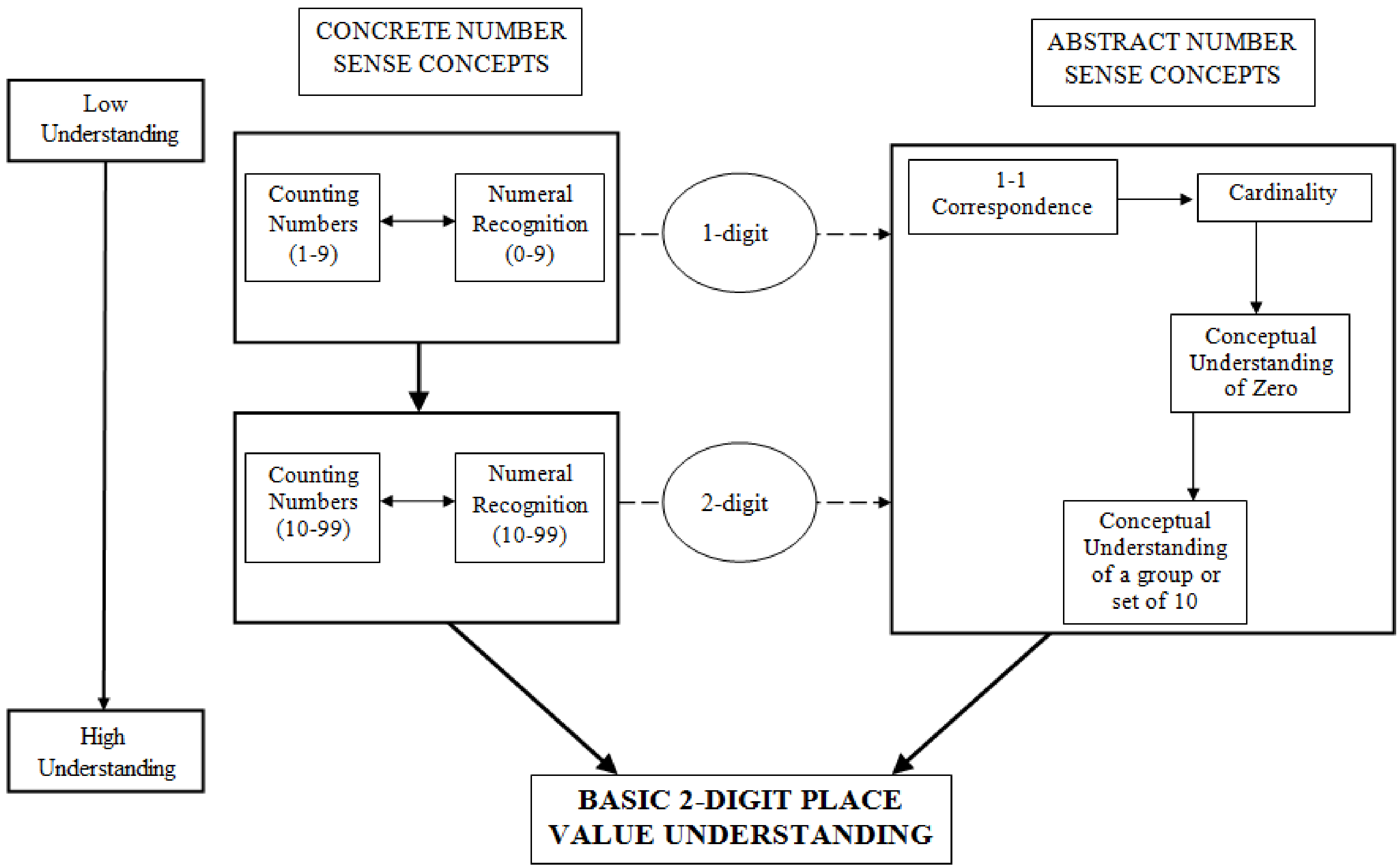


Figure 2

MTP Math Number Chart Including Color-Coded ten-frame Representations of the Numerals 0-39

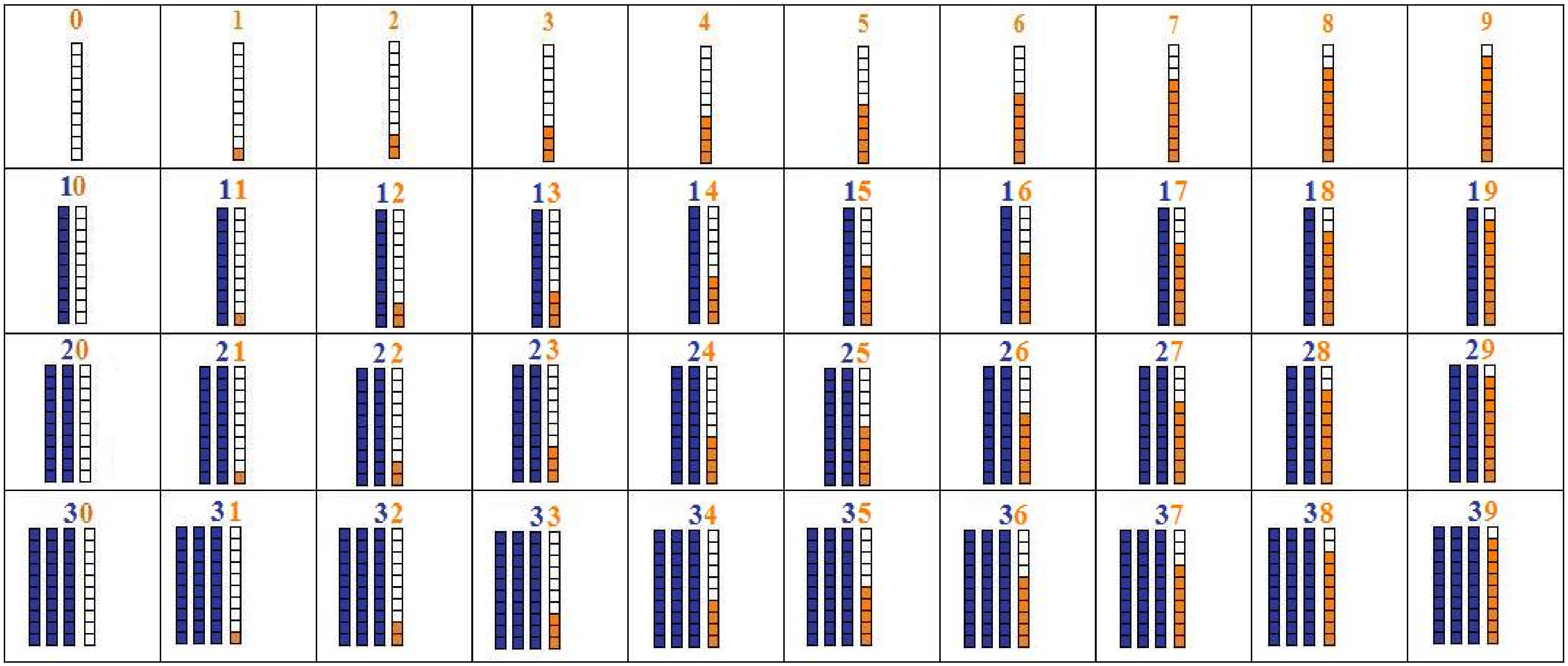


Figure 3

The Three Basic Numerical Representations of the Number 15

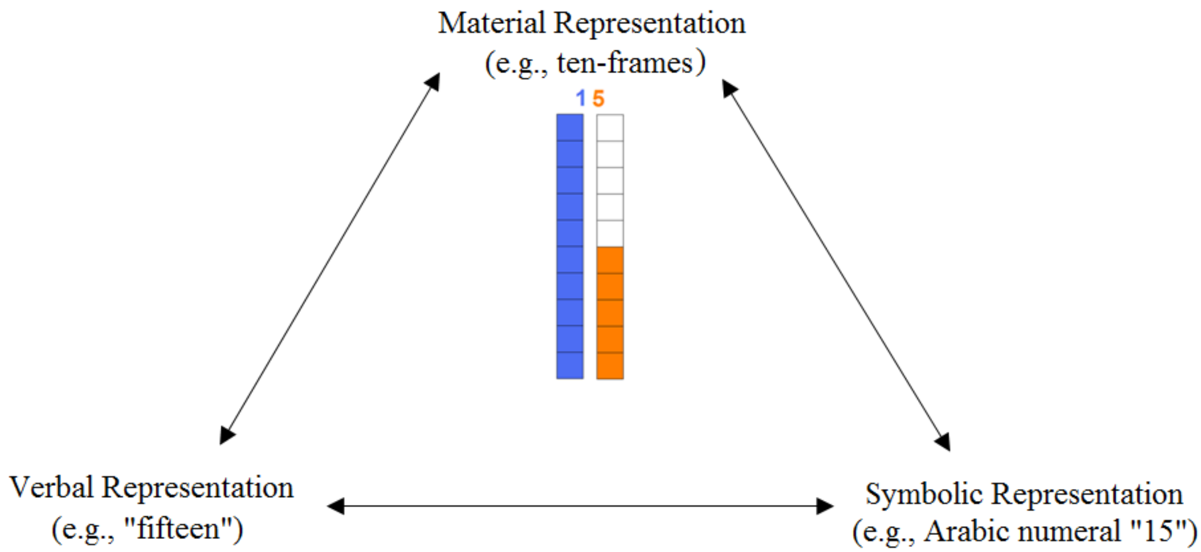


Figure 4

Flow Chart Illustrating the Prompts and Scoring Procedures for a Sample Item from the Place Value Portion of the Dynamic Assessment

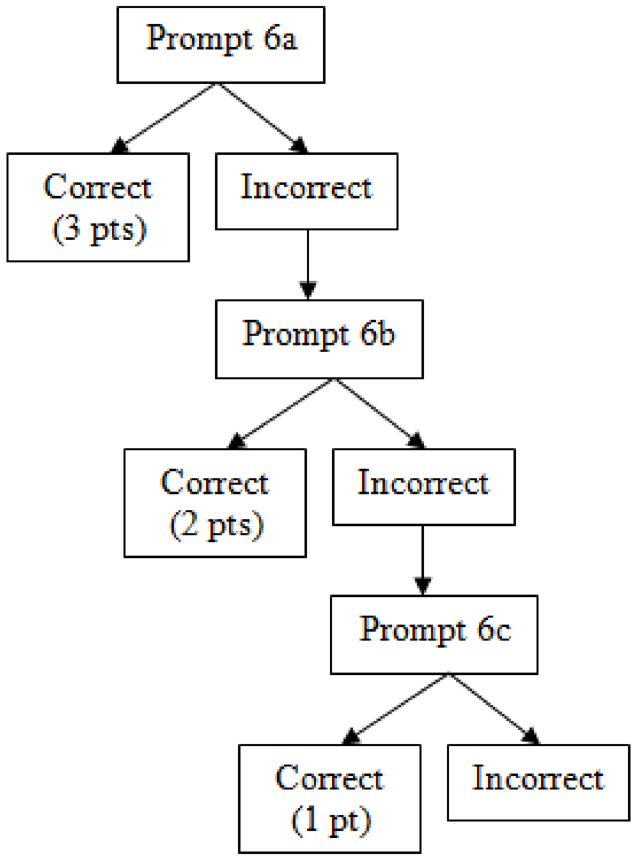


Figure 5

Sample Item Displaying Visual Representations of Ten-frames, Initial Question, and Three Successive Scaffold Prompts from the Number Sense Test

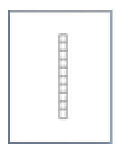
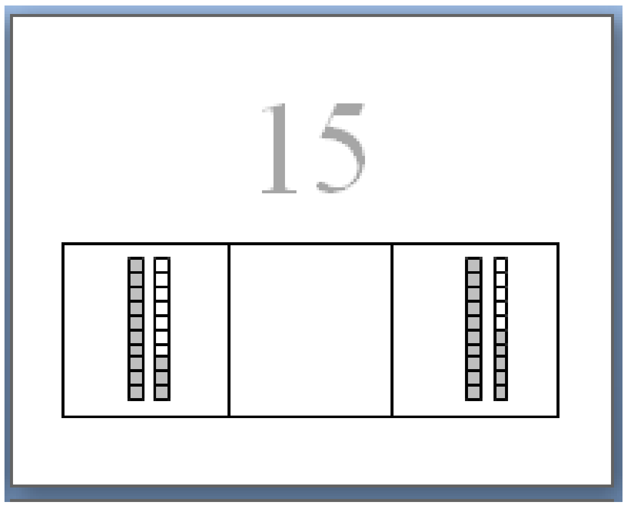
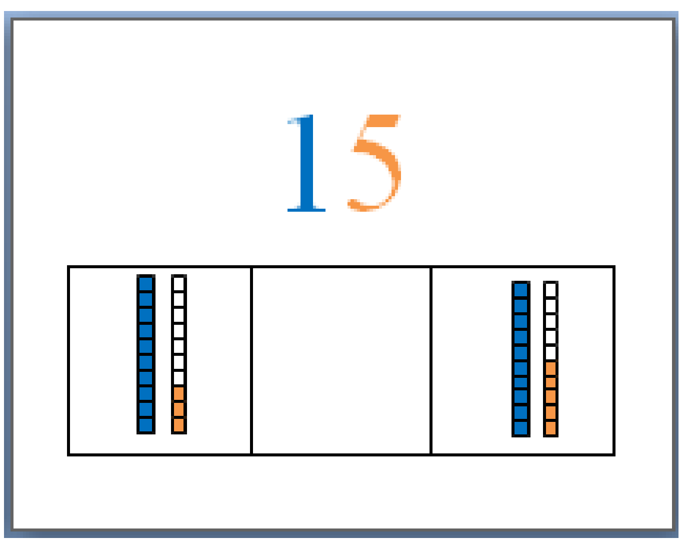
3. Given 8 teddy bear counters:
a) *"How many teddy bear counters are there?"* >> If successful, go to (3e) >> If incorrect or no response, go to (3b)
b) "Count the teddy bears to tell how many there are." >> If successful, go to (3e) >> If incorrect or no response, go to (3c)
c) "*Use this 10-frame to help you count teddy bears to tell how many. Put one bear in each square as you count.*"  >>If successful, jump to (3e) >>If incorrect or no response, go to (3d)
d) *"Let's count the teddy bear counters together to tell how many."* >>Count the entire number of teddy bears together, then go on to (3e)

Figure 6

Sample Item Displaying Visual Representations of Ten-frames, Initial Question, and Two Successive Scaffold Prompts from the Place Value Test

Question Format and Visual Representation
Initial Question: Can you point to the ten-frame picture that matches the number? 
Scaffold 1: Maybe seeing the ten-frame pictures in different colors will help you. How about now? 
Scaffold 2: Let's count the colored squares together in the ten-frames to find out which is the right number.

Appendix A: Overview *MyTeachingPartner Math*

Place Value Activities and Lesson Objectives

Activity	Lesson Objective(s)	Overview
Jan Week 4 *Number Chart 10 & 11*	Recognize Numerals 0-11	Students explore the visual representations of numbers presented in ten-frames on the Number Chart. This lesson provides students with the first introduction to two-digit numbers and encourages them to create groups of ten. Instruction is anchored around 10 for the entire lesson.
Feb Week 4 *Anno's Counting Book*	Recognize numerals 0-12 Count with objects up to 12 (place value: groups of 10 and 2)	Following the reading of *Anno's Counting Book* (by Mitsumasa Anno), students use snap cubes to build up to the number 12. Along the way, discussion focuses on groups of ten and connections are made to the Number Chart. This activity provides students with a visual representation of how two-digit numbers are created, and emphasizes the place value concepts of groups of ten and the ones that are leftover.
March Week 1 *Building Numbers*	Recognize numerals 0-15 Count with a running start 2-15 Count on from 2-15 Identify the number that comes before and after, to 14	Students work with the Number Chart and ten-frame representations for the numbers 13, 14, and 15. Emphasis is placed on observing patterns in the rows and columns of the chart. Students work with the teacher to add new numbers to the chart, making predictions about what number will come next and how many ten frames/leftovers each number will have.
Apr Week 3 *How Many Hands*	Recognize patterns in the Number Chart	This activity closely relates to the patterns discussed in Mar Week 4. Students count the number of hands of students in their small group, and find that amount on the Number Chart. Extension questions for this activity focus on changing the number of hands in the set by ten (more or less) and what patterns can be observed on the Number Chart in relation to ten.
May Week 1 *Number Chart Matching Game*	Recognize numbers 0-39	The culminating place value and number recognition activity for the year. Students play a number matching game, matching numerals 0-39 to ten-frame representations on Number Chart. The discussion within this activity is oriented around a summary of a two-digit place value, reminding students that the first number tells how many tens there are and the second number tells how many ones there are.